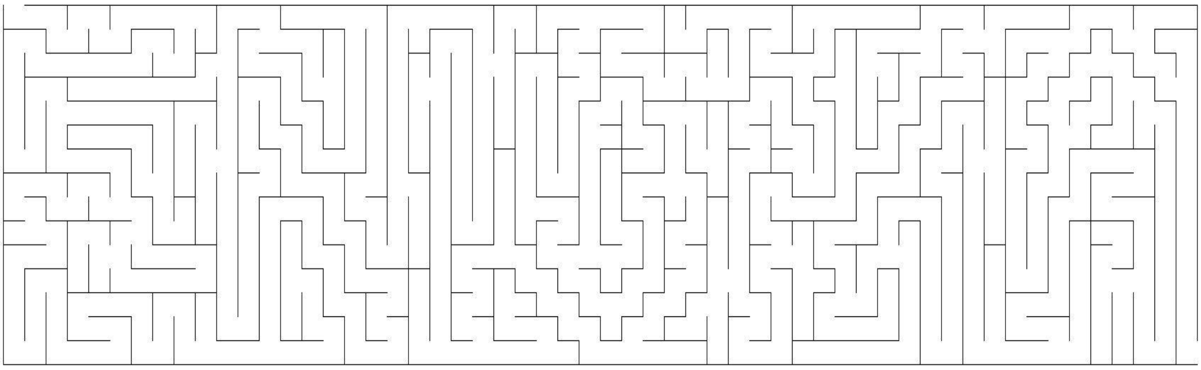
Enigma:
The Newsletter of PuzzleSIG



ENIGMA

248

(ROMFORD MARKET TO MOOR LANE)

'Mathematical problems, or puzzles, are important to real mathematics, just as fables, stories, and anecdotes are important to the young in understanding real life'.

Terry Tao

March 2021

Electronic Version of this Newsletter
Email enigma.mensa@yahoo.co.uk and I'll send you a copy

About Enigma
Enigma is the newsletter of Puzzle SIG.
The SIG for anyone interested in puzzles. The scope covers word puzzles, crosswords, logic puzzles, Japanese puzzles, mathematical brain teasers, lateral thinking problems, quizzes and picture quizzes, discussion of physical / mechanical puzzles, computer / internet based puzzles and puzzle games, and puzzle books and publications, and experimentation and innovation of new puzzle forms.

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How to Join
You can join Puzzle SIG by visiting mensa.org.uk/sigs (member login required).

Welcome to Enigma 248

Hello and welcome to another issue of Enigma.

Many thanks to Brooks Rimes and Christa Ramonat for their contributions, which were both of a wordy nature. Nearly all of the other puzzles are from elliottline.com and are of a more mathematical flavour, which is great if you like those sorts of puzzles, but a pity if you don't. They just happen to be the type that I personally mostly seem to create.
Remember that this is your newsletter and will thrive on more and varied contributions from its readers.
One thing I keep meaning to do, but haven't quite got around to yet, that might help broaden the scope of Enigma, is to write an article or three about twisty puzzles. Just last year in lockdown 1.0 I decided to try to solve the Rubik's Cube, and from that I have developed a great love for a variety of what are collectively called twisty puzzles: 3x3x3, 2x2x2, 4x4x4, 5x5x5, Mirror Cube, Kilominx, Pyramorphix, the list is extensive. Look forward to hearing more about them in future issues.

Please try all of the puzzles, and if you have any feedback, let me know and I'll pass it on to the puzzle's creator.

As ever, if you get stuck and need a hint, drop me a line and I'll be happy to help.
Please keep your puzzles, answers, comments, queries, suggestions, etc coming in.

Happy puzzling
Elliott.

247.01 - COMPETITION: Cube - Elliott Line

192.
The formula is $x^3(1 + 2\pi\sqrt{2}/3 - 5\pi/4)$, where x is the cube side length.

WELL DONE TO:
Abhilash Unnikrishnan
Paul Clark
Stuart Nelson
Karl Billington
Paul Bostock
Alan Routledge
Jon Abbott
Jeff Jowers
Roisin Carters

248.01 - COMPETITION: Powers of 2 - Elliott Line

I have created a series of numbers, each being the concatenation of the powers of 2, beginning with 2:

2, 24, 248, 24816, 2481632, 248163264 etc.

When you take a term from this series and divide by the final power of 2, you get another whole number, for instance 24816 divided by 16 is 1551. In the case of 248 (the number of this issue of Enigma), you get 248 / 8 = 31, which is prime.

Which other term from this series, when you divide by the rightmost power of 2, will result in a prime number?

This is a competition, but not for prizes, only bragging rights. Every correct answer I receive will get an honourable mention in the next issue of Enigma. Send your answer to me at enigma.mensa@yahoo.co.uk .

248.02 – Three Hundred and Eighty Something – Elliott Line

A few special numbers can be expressed as the product of a set of three or more integers in arithmetic progression. For instance 2x5x8 = 80, 3x4x5x6x7 = 2520, 4x6x8x10 = 1920. Of the three-digit numbers that starts with 38, TWO are those special numbers. Which ones, and how?

248.03 – Base 4 Code – Elliott Line

I have taken some quotations, and I have replaced each of the letters with the numbers that denote their position in the alphabet. However, I have used the base 4 number system.
Be careful, as some sequences of numbers could lead to several words, for instance 31110 could mean CAT (3,1,110), but could equally mean MAD (31,1,10).

	1 = A	2 = B	3 = C
10 = D	11 = E	12 = F	13 = G
20 = H	21 = I	22 = J	23 = K
30 = L	31 = M	32 = N	33 = O
100 = P	101 = Q	102 = R	103 = S
110 = T	111 = U	112 = V	113 = W
120 = X	121 = Y	122 = Z	

11020213213103 1133310223 33111110 211103110 1233102 110203310311 1132033 3112311 1102011 211103110 3312 1102011 1131121 11020213213103 1133310223 33111110. 22332032 1133333101132.

12133111 3132 1112332110 1021113021110121, 2111110 12133111 3132'110 1112332110 1102011 3333210311101111132311103 3312 1112332110213213 1021113021110121.

248.04 – Divisible by 32 – Elliott Line

If n can be any natural number (positive whole number), when is (26^n + 6^n) NOT divisible by 32?

248.05 – Change a Letter – Elliott Line

Change one letter from each word, and THEN re-space to form an aphorism:

For example:
TO YOGA INFO ILL GRIN FAN LIE TIER
becomes:
TR YAGA INFA ILA GAIN FAI LBE TTER
then:
TRY AGAIN FAIL AGAIN FAIL BETTER

CHEF CAME TO IT BUT NOT WISE AT BRAG AT OUR ASH AS FAST ONE

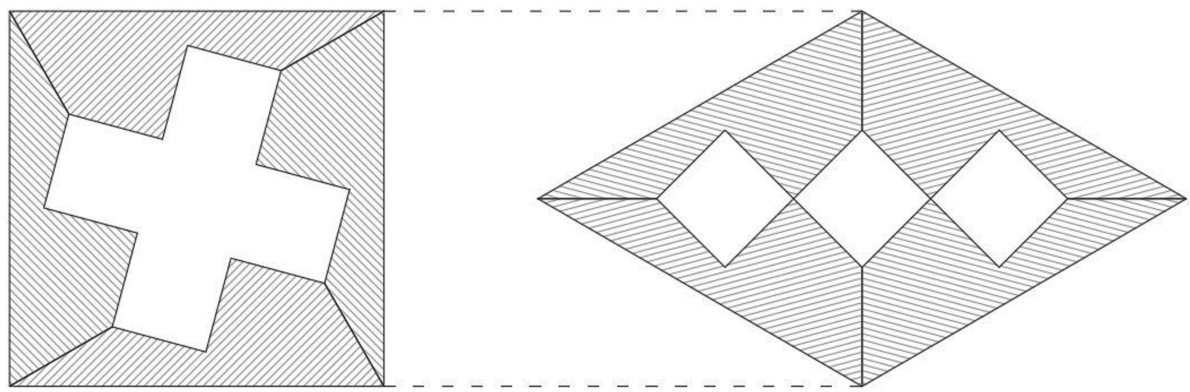
248.06 – Circular Table – Elliott Line

A group of 21 friends are seated around a large circular table. By a strange coincidence, the sum of the ages of ANY six consecutively seated friends adds up to 200. If the person at seat 1 is aged 25 and the person and seat 8 is aged 33, how old is the person at seat 15?

248.07 – Dissection – Elliott Line

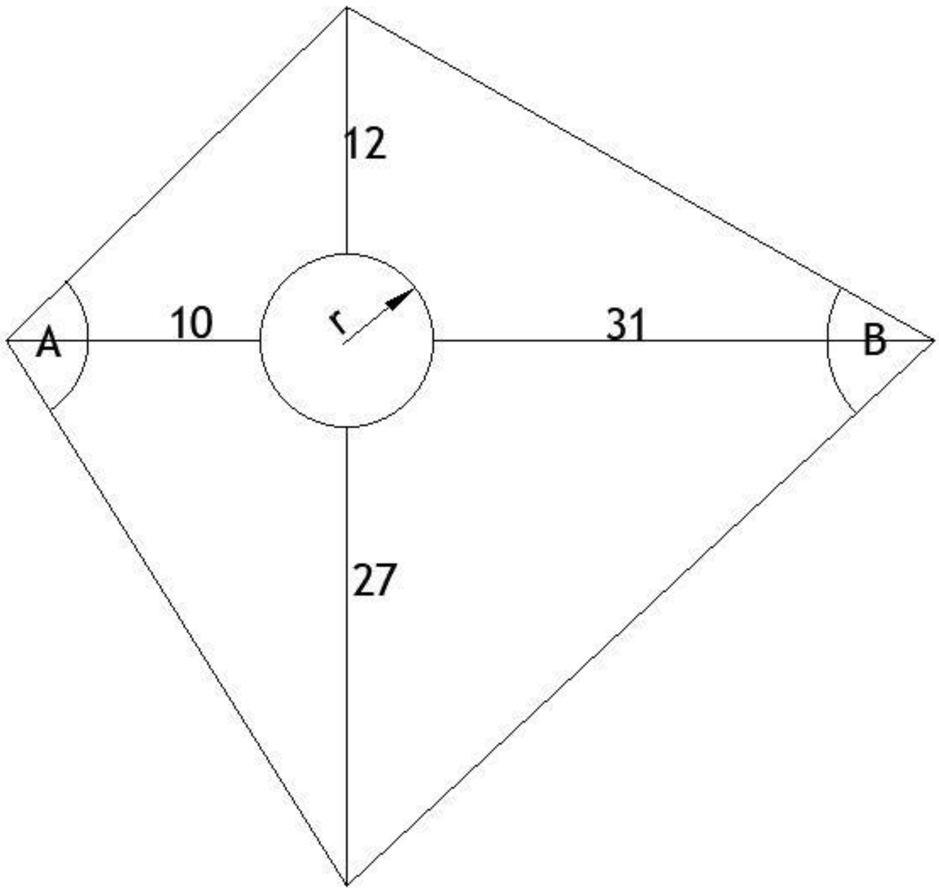
Four identical pieces forming a square with a cross-shaped hole can be rearranged (after flipping half the pieces) into a rhombus shape with three square holes. The shorter diagonal of the rhombus is the same as the side length of the square.

What proportion of the square is shaded?



248.08 – Compass Points – Elliott Line

Four lines of lengths 12, 31, 27 and 10 are drawn respectively from the North, East, South and West points on a circle, heading directly away from the circle's centre as shown.



The four endpoints are joined with straight lines to form an irregular quadrilateral. Two opposite angles of this quadrilateral, A and B, add to 180 degrees.

What is the radius, r, of the circle?

248.09 – Cryptogram – Brooks Rimes

Here is a cryptogram from Cryptograms - Quotes of Famous Women by Brooks Rimes, available on Amazon at https://www.amazon.com/dp/1696947170.

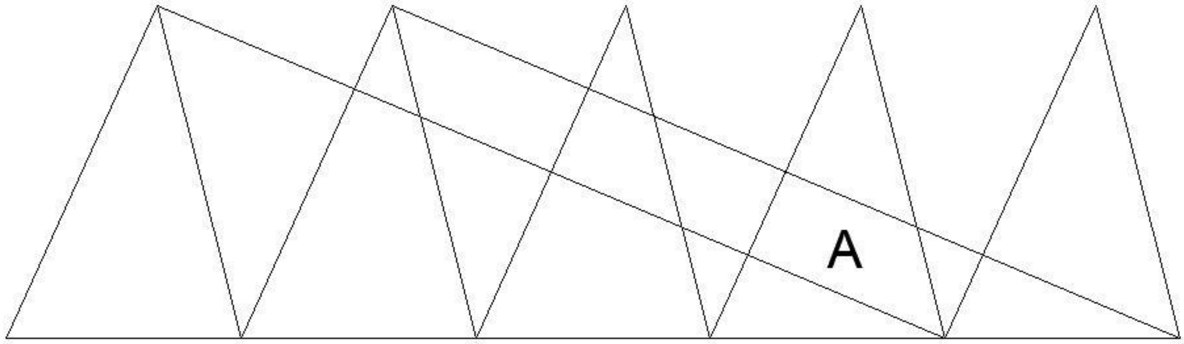
LD SPMC ZG EZZP OZLNCED SV YRM OZLNCED ZG OUMIMT,

HMU U-SFGZTLMP NMZNUM HRZ RCIMC ETMCY PMCU ZG

OZEIMIVCYSZF; YRCY SV HRCY S OCUU EZZP OZLNCED.

- WCFM CAVYMF

248.10 – Five Triangles – Elliott Line



Five identical triangles are placed in a row with their bases collinear as shown. A couple of diagonal lines are drawn from the apexes of some triangles to the lower vertices of others. If each of the five triangles has an area of 1, what is the area of the region marked with an A?

248.11 – Mental Calculation – Elliott Line

Calculate the value of the following without electronic assistance:

$$\frac{(3-\sqrt{5})\sqrt{(3\sqrt{5}+7)}}{\sqrt{2}}$$

248.12 – Round Table – Elliott Line

A family that consists of parents and 6 children sit around a table in age order: father, mother, eldest child etc, so the youngest child sits next to the father.
The gender of each of the children is male or female, with 50% probability of each.

Knowing only this information, what is the probability that all the males are seated together?

248.13 – Nine Letter Take-aways – Christa Ramonat

There has been a riddle circulating that goes like this "What is the one word in the English language where you can take away one letter at a time and it still remains a word each time you remove a letter?". An answer is provided, claiming to be the only such word. If you have not seen this, the answer is at the end of this issue. I figured there had to be more words that would meet these criteria so every time I was walking the dog alone, having trouble sleeping or waiting for something I thought up possibilities until I came up with a list of 180 words. All of these words and the words made from them passed a screen for legal scrabble words. Of these, 64 passed a more rigorous screen that used a number of other sources in addition to scrabble. I learned a lot of new words in the process which should make me a better scrabble player. Some people think this is fun challenge, others think it is interesting but have no idea how to start and some do not like puzzles at all. Like any puzzle, for example, Sudoku, it is difficult at first but after a while you learn tricks and the puzzles get easier. One of the tricks I found was to work out from the middle, focusing on a five-letter word and then working upward and downward from there.
So, if you are looking for a new challenge to exercise new pathways in your brain, give this a try. I have provided a few of my favourites at the end the issue, and can provide my full list upon request!

248.14 – Special K – Elliott Line

K, L and M are all positive whole numbers.

For the certain special values of K that we seek, the same values of L and M that cause (KxL)+(4xM) to be a multiple of 11 also cause (KxM)+(5xL) to be a multiple of 11.

For instance, K ISN'T 2, because some values of L and M that make (2xM)+(5xL) a multiple of 11 (eg L=1,M=3) when you plug those same values of L and M into (2xL)+(4xM) give a number that is NOT a multiple of 11 (in this case 14).

Out of the possible values of K for which the divisibility by 11 of (KxL)+(4xM) and (KxM)+(5xL) are always in agreement, what number is the THIRD LOWEST PRIME?

248.15 – Winner Stays On – Elliott Line

Four friends, Alfie, Billie, Charlie and Debbie, play a series of games on their pool table. At each point, two of the friends are playing each other while the other two are reduced to spectating. After each game, the winner stays at the table and will go on to play whichever of the two spectators has been waiting the longest since their last game, and the loser becomes a spectator for the next game, in order to ensure everybody gets to play.

After they have finished Alfie has played in eight of the games, Billie three, Charlie six and Debbie five.

Who lost in the ninth game?

248.16 – Power Play – Elliott Line

What is the remainder of (10^101 + 11^101) when divided by 21?

248.17 – Special Sequence – Elliott Line

This is a special sequence:

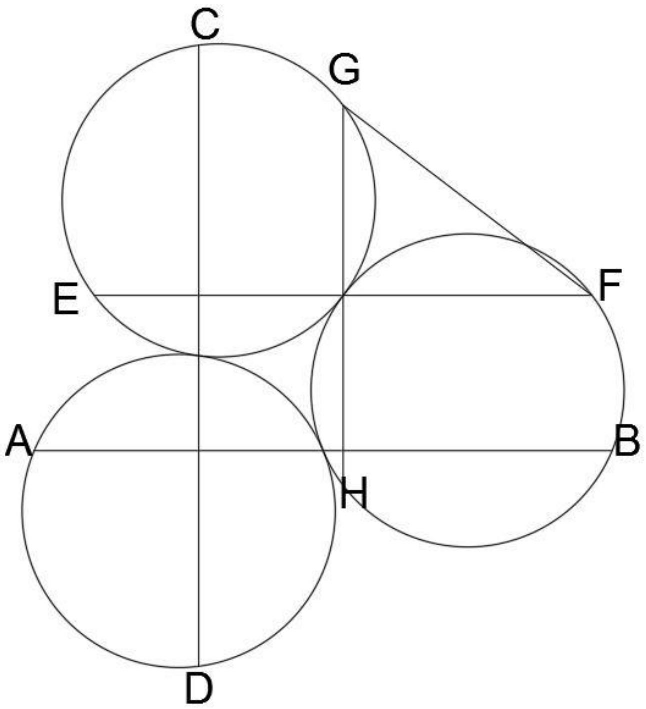
3 1 2 1 3 2

For each number 'k' from 1 to 3, the number of numbers between the pair of 'k's is equal to k.
In other words, there is one number between the pair of 1s, two numbers between the pair of 2s and three numbers between the pair of 3s.

Can you create a similar sequence containing pairs of numbers but starting from 4 and using all numbers up to n (n can be whatever you need it to be to make a sequence that has no gaps), where for each number k, the number of numbers between the pair of 'k's is k (so there will be four numbers between the pairs of 4s, five numbers between the pair of 5s, etc.)?

248.18 – Three Tangent Circles – Elliott Line

Three unit radius circles are arranged to each be tangent to the other two. Four lines: AB CD EF GH are drawn through these tangent points as shown, extending both ways to meet the circles again, with AB and EF drawn horizontally and CD and GH drawn vertically. What is the length of FG?



***** SOLUTIONS *** SOLUTIONS *****

248.02 – Three Hundred and Eighty Something – Elliott Line
The more obvious solution is that 2 x 4 x 6 x 8 = 384.
The second is a little bit sneaky, as I didn't specifically mention you could use negative numbers (as long as the result is positive): -7 x -1 x 5 x 11 = 385.

248.03 – Base 4 Code – Elliott Line
Things work out best for those who make the best of the way things work out. John Wooden.
You can avoid reality, but you can't avoid the consequences of avoiding reality.

248.04 – Divisible by 32 – Elliott Line
n=2
Since 26 and 6 are both even, if n is greater than or equal to 5, both parts will independently be divisible by 32, so be only need to consider n = 1,2,3,4.
When n=1, it is obvious that 26+6=32.
When n=3, we note that (x^3 + y^3) can always be factorised into (x+y)(x^2-xy+y^2), so letting x=26 and y=6 shows that this is divisible by 32.
When n=4, each part will be an odd number multiplied by 2^4, so will naturally have a remainder of 16 when divided by 32. Putting both of them together gives a multiple of 32.
That just leaves n=2. Unlike all of the other numbers, there is no inherent reason why this would be divisible by 32, and simply checking shows that it isn't.

248.05 – Change a Letter – Elliott Line
CHEF CAME TO IT BUT NOT WISE AT BRAG AT OUR ASH AS FAST ONE
THEF LAME TH AT BUR NST WICE AS BRIG HT BUR NSH AL FASL ONG
THE FLAME THAT BURNS TWICE AS BRIGHT BURNS HALF AS LONG

248.06 – Circular Table – Elliott Line
Since every six consecutive friends adds to the same amount, it follow that 1-6 totals the same as 2-7, and since these two groups have five people in common it follows that 1 is the same age as 7. By the same reasoning, every person is the same age as the person 6 seats away. So 1, 7, 13 and 19 are all the same age. But the group (19,20,21,1,2,3) has the same total as (20,21,1,2,3,4), so 19 and 4 are the same age. Following this through the ages repeat in a pattern every three people, and each consecutive group of three people must add up to 100.
We are given the ages of 1 and 8 and asked for the age of 15. But 8 is the same age as 2, and 15 is the same age as 3, and together they will form a consecutive group of three adding to 100. If the first two are aged 25 and 33, the third must be aged 42.
The person seated at position 15 is aged 42.

***** SOLUTIONS *** SOLUTIONS *****

248.07 – Dissection – Elliott Line

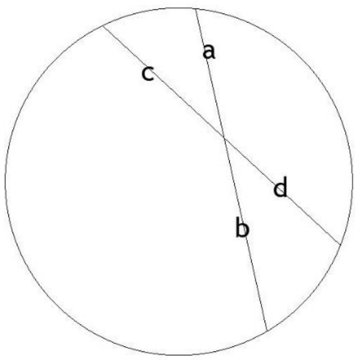
The rhombus is the shape of two equilateral triangles placed together, and therefore its long axis will be sqrt(3) (assuming the side length is 1 unit).
The area of the square is 1 and it is missing 5 square shapes.
The area of the rhombus is sqrt(3)/2 and it is missing 3 square shapes.
Since the two shapes are made up of the same pieces their areas must be identical, so (letting x = the area of 1 small square):

$1 - 5x = \text{sqrt}(3)/2 - 3x$
$1 - \text{sqrt}(3)/2 = 2x$
$x = (1 - \text{sqrt}(3)/2)/2$

The proportion of the square that is shaded is 1 – 5x, which is approximately 66.5%

248.08 – Compass Points – Elliott Line

The fact that opposite angles of the quadrilateral add to 180 degrees tells us that the quadrilateral is cyclic, ie that a circle drawn through any three vertices will also pass through the fourth.
The fact that the four vertices can lie on a circle lets us use a fact about lines crossing in circles: the product of two opposing distances from a particular point within a circle to the edge of the circle will always be the same, regardless of the angle of the line through the point.
In the following diagram for instance, ab = cd, as a general rule.



We can use this fact if we take the crossing points as the centre of the circle, and the four distances as (12+r), (31+r), (27+r) and (10+r).

$(12+r)(27+r) = (10+r)(31+r)$

$324 + 39r + r^2 = 310 + 41r + r^2$
$14 = 2r$
$r=7$

So the radius of the circle is 7.

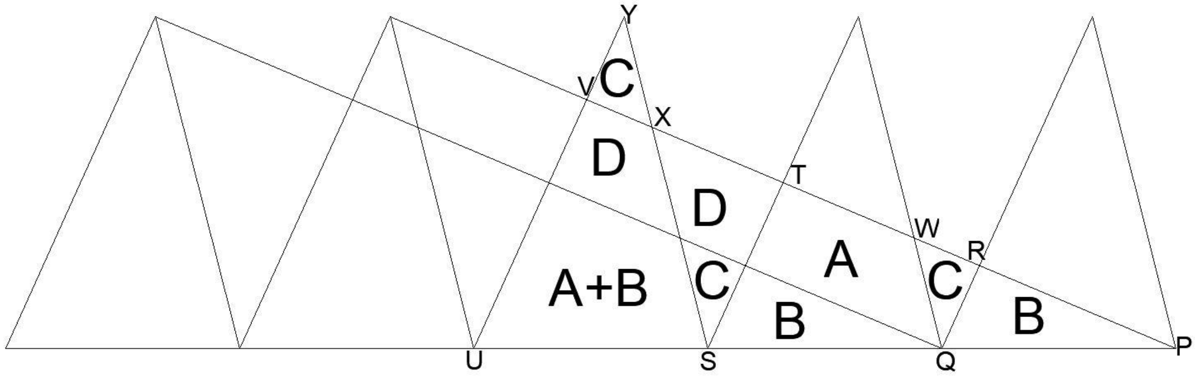
***** SOLUTIONS *** SOLUTIONS *****

248.09 – Cryptogram – Brooks Rimes

My idea of good company is the company of clever, well-informed people who have a great deal of conversation; that is what I call good company.

- Jane Austen, English Novelist

248.10 – Five Triangles – Elliott Line



Because of the symmetry of the set-up we are able to determine a few facts, irrespective of the particular shape of the triangle. We are able to assign areas A, B, C, D to different regions as shown. From the triangle SUY, A+B+C+D=1.

We can identify similar triangles PQR and PST, whose linear scaling is 1:2 and so whose area scaling is 1:4. So:

4B = A+2B+C

Similarly PUV has an area nine times that of PQR, so:

9B = 2A+2C+3B+2D

The first leads to 2B = A+C and the second leads to 3B = A+C+D, therefore B=D. Then, since B+D = A+C and all four add to 1, it follows that B and D are each equal to 1/4.

Next, look at similar triangles STX and QRW. The former is twice the linear scale, so four times the area of the latter, so:

C+D = 4C

3C = D

But we know that D=1/4, so C=1/12.

Since A+B+C+D=1, A = 1-1/4-1/12-1/4 = 5/12.

The area A is equal to five twelfths.

***** SOLUTIONS *** SOLUTIONS *****

248.11 – Mental Calculation – Elliott Line

$$\frac{(3-\sqrt{5})\sqrt{(3\sqrt{5}+7)}}{\sqrt{2}} = x$$

NOTE THAT, SINCE $3 > \sqrt{5}$, THE VALUE OF x WILL BE POSITIVE.

SQUARE BOTH SIDES:

$$x^2 = \frac{(3-\sqrt{5})^2(3\sqrt{5}+7)}{2}$$

EXPAND THE SQUARE OF $(3-\sqrt{3})$

$$x^2 \quad \frac{(9-6\sqrt{5}+5)(3\sqrt{5}+7)}{2}$$

TAKE OUT A FACTOR OF 2 FROM THE FIRST BRACKET TO CANCEL THE 2 BELOW:

$$x^2 = (7-3\sqrt{5})(7+3\sqrt{5})$$

MULTIPLY THE BRACKETS TOGETHER:

$$x^2 = 49 - 9 \times 5 = 49 - 45 = 4$$

SINCE WE KNOW x IS +VE IT IS THE +VE SQUARE ROOT OF 4.

$x = 2$, AND THAT IS THE ANSWER!

248.12 – Round Table – Elliott Line

There are 64 (2^6) possible gender permutations of the six children. For all the males to be seated together, all of the girls need to be older than all of the boys, and there is one such arrangement for each of the possible numbers of girls from 0 to 6, giving 7 possible valid arrangements.
Therefore the probability is 7/64, which is just under 11%.

248.13 – Nine Letter Take-aways – Christa Ramonat

The "only" word in the English language is STARTLING (STARLING/STARING/STRING/STING/SING/SIN/IN/I)
A couple of my favourites include
RESTARTED (RESTATED/RESTATE/ESTATE/STATE/STAT/SAT/AT/A)
SHOWERING (SHOWRING/SHOWING/SOWING/OWING/WING/WIN/IN/I)
GRANDEURS (GRANDEUR/GRANDER/GRADER/GRADE/GRAD/RAD/AD/A)
And my best one is
BRAINIEST (BRINIEST/BRINIES/BRINES/BRINE/BRIN/BIN/IN/I)

***** SOLUTIONS *** SOLUTIONS *****

248.14 – Special K – Elliott Line

The multiples of 11 will coincide whenever (K^2) – (4 x 5) is a multiple of 11. This occurs when K is either 3 more or 3 less than a multiple of 11, so 3, 8, 14, 19, 25, 30 etc.
The first three times this is a prime number are 3, 19 and 41. Therefore the answer is 41.

For an explanation of why this is the case, read on:
Under the assumption that KM+5L and KL+4M are both congruent to 0, modulo 11, we can multiply either side by an integer and that congruence will be preserved. Therefore we can multiply the first expression by K (which is defined as an integer) and the second by 5. Therefore K^2.M+5LK and 5LK+20M are also both 0 mod 11. Subtracting one expression from the other unifies the two expressions but maintains the divisibility by 11. Doing so makes the 5LK terms cancel out so we end up with (K^2 – 20)M is 0 mod 11. Therefore for our purposes, where we are interested in the special values of K, this occurs when K^2 – 20 is a multiple of 11, which is where we came in.
As a side note, if you're wondering what happens if you follow the other strand and assume M is a multiple of 11, it's not difficult to show that that means that L must also be a multiple of 11 for either/both of the original expressions to be a multiple of 11, which it will be for all K.

248.15 – Winner Stays On – Elliott Line

At first sight the question seems impossible. There is definitely not enough information to decide who plays who and when but, believe it or not, there in enough information to answer the particular question of who lost in the ninth game.
Firstly lets see how many games there were altogether. To do this we an simply add together the number of games played by each individual, and divide that total by two, since there are two players in each game. This tells us that there were 11 games in total. Because of the way they have of rotating players, even if a player loses, they will be back at the table three games later. The key now is to look at Billie, who has played the least games. If we assume they lost every game they played they would sill play roughly a third of the games. Say Billie played in game 1, they would play again in game 4, game 7 and game 10, but that is too many games. Similarly if they played and lost in game 2, they would reappear for game 5, game 8 and game 11. Still too many. The only possibility is that Billie played and lost in game 3, game 6 and game 9.
Billie lost in the ninth game.

***** SOLUTIONS *** SOLUTIONS *****

248.16 – Power Play – Elliott Line

The remainder is zero. In fact 10^n + 11^n is divisible by 21 whenever n is an odd number.
You can prove this by showing that when n is odd, (a + b) is a factor of (a^n + b^n).
You can factorise (a^n + b^n) as:
(a + b)(a^(n-1) – a^(n-2)b + a^(n-3)b^2 - … - ab^(n-2) + b^(n-1))
When you multiply the massive second bracket by (a + b) almost everything cancels out, leaving only (a^n + b^n). Here is a specific example using n=5:

(a+b)(a^4 – a^3.b + a^2.b^2 – a.b^3 + b^4)
Multiplying each term in the second bracket first by a:
(a^5 – a^4.b + a^3.b^2 – a^2.b^3 + a.b^4)
And then by b:
(a^4.b – a^3.b^2 + a^2.b^3 – a.b^4 + b^5)
Adding those two brackets leaves only the first and last terms:
(a^5 + b^5)

So as long as a, b and n and integers, the whole second bracket will be too. Therefore (10 + 11) divides into (10^101 + 11^101).

248.17 – Special Sequence – Elliott Line

One possible answer is:
9 10 11 12 4 5 6 7 8 4 9 5 10 6 11 7 12 8
The structure hints at a general method (which might not have been apparent from the original example, but nevertheless follows the same idea). Using numbers m to n, n needs to be (at least) 3m.

248.18 – Three Tangent Circles – Elliott Line

The length of FG will be 2.