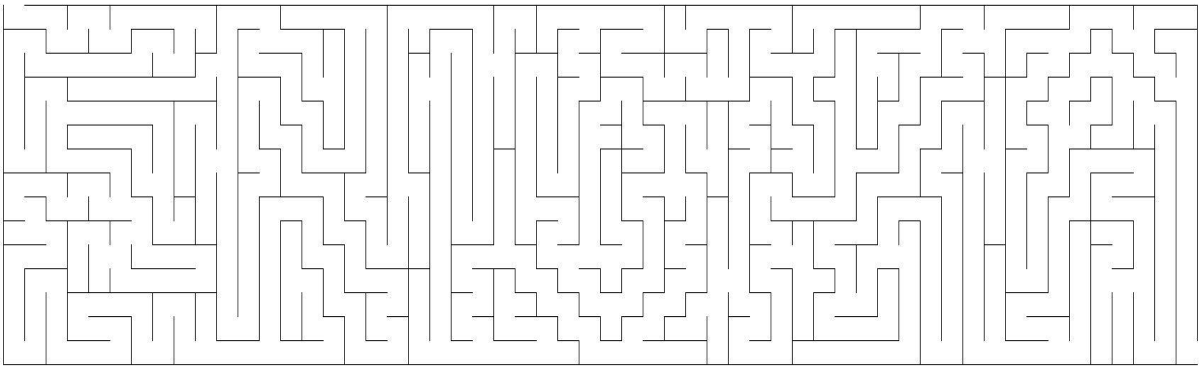
Enigma:
The Newsletter of PuzzleSIG



ENIGMA

249

'Mathematical problems, or puzzles, are important to real mathematics, just as fables, stories, and anecdotes are important to the young in understanding real life'.

Terry Tao

June 2021

Electronic Version of this Newsletter
Email enigma.mensa@yahoo.co.uk and I'll send you a copy

About Enigma
Enigma is the newsletter of Puzzle SIG.
The SIG for anyone interested in puzzles. The scope covers word puzzles, crosswords, logic puzzles, Japanese puzzles, mathematical brain teasers, lateral thinking problems, quizzes and picture quizzes, discussion of physical / mechanical puzzles, computer / internet based puzzles and puzzle games, and puzzle books and publications, and experimentation and innovation of new puzzle forms.

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How to Join
You can join Puzzle SIG by visiting mensa.org.uk/sigs (member login required).

Welcome to Enigma 249

Hello and welcome to another issue of Enigma.

Thanks to the contributors to this issue: Brooks Rimes, Rosemary Hodgson and Graham Holmes, providing a bit of variety beyond the usual 'Elliott Line' brand of mathematical problems. Brooks provides a great Alphacipher, which is a lot of fun to unpick; Rosemary brings us a verse which has been split apart in an unusual way and requires reassembling; and Graham delivers an intriguing trivia quiz of words to try to find. You'll also find a couple of logic puzzles of my devising: Shadowbox (if you enjoy it you might enjoy my book full of them, just search online), and Galaxy Quest. Also, as promised in the last issue, I have written a short article about the Rubik's Cube, which I hope you'll find interesting and illuminating, despite its conspicuous lack of pictures!

Please try all of the puzzles, and if you have any feedback, let me know and I'll pass it on to the puzzle's creator.

As ever, if you get stuck and need a hint, drop me a line and I'll be happy to help.
Please keep your puzzles, answers, comments, queries, suggestions, etc coming in.

Happy puzzling
Elliott.

248.01 - COMPETITION: Powers of 2 - Elliott Line

2481632 / 32 = 77551.

WELL DONE TO:
Johann Muller
Vladimir Turek
Pat McNally
Matthew Francis
Agnijo Banerjee
Christa Ramonat
Colin Packer
Paul Clark
Karl Billington
Roisin Carters

249.01 - COMPETITION: Fun with Resistors - Elliott Line

If you combine resistors in series, you add their resistances together. For example 10 ohms and 20 ohms combined in series gives 30 ohms.

If you combine resistors in parallel, you take the reciprocal of the sum of reciprocals of the individual resistors. For example 12 ohms and 24 ohms combined in parallel gives you 8 ohms (1/12 + 1/24 = 1/8).

You have a supply of 120 ohm resistors, how can you use just twelve of them to achieve a resistance of 249 ohms (249 being the number of this issue of Enigma)?

This is a competition, but not for prizes, only bragging rights. Every correct answer I receive will get an honourable mention in the next issue of Enigma. Send your answer to me at enigma.mensa@yahoo.co.uk .

249.02 – Alien Number System – Elliott Line

Imagine a number system where the only numbers are those that are 1 greater than a multiple of 20, for instance, 21, 81, 1741. You cannot add or subtract using this number system without the result being a number outside the number system, however it is possible to multiply, as multiplying together two numbers that are each 1 greater than a multiple of 20 will result in a third number that is also 1 greater than a multiple of 20. For example, 21 x 61 = 1281.
'Prime' numbers exist in this system, defined as numbers that cannot be formed by multiplying together two smaller numbers in the number system. All actual primes, such as 41, are obviously still prime in this system, but other numbers, such as 21 or 81, are not prime usually, but are in this system.
One well known fact about ordinary numbers is that they are the product of prime numbers in exactly one way, for example 72 = 2x2x2x3x3. However, it is possible for numbers in this special number system to be the product of 'prime' (within the system) numbers in more than one way.
What is the smallest number in this number system that is the product of 'primes' in two distinct ways?

249.03 – Always True? – Elliott Line

I happened to notice that if I took a number that was the sum of two different squares (eg, 4+1 = 5) and multiplied it by a different number that was also the sum of two different squares (eg, 9+1 = 10), the result (50) would also be the sum of two different squares: (49+1).
But is this always true?

249.04 – Climb the Ladder – Elliott Line

A while ago I 'invented' an interesting mathematical algorithm as follows.

Start with any whole number 'n'.
If 'n' is prime, stop.
If 'n' is composite, list its prime factors and find the largest, let's call it 'm'
Let your new 'n' be equal to n+m+1, and repeat the whole sequence.

For instance if we start with 15:

15
Not prime, highest prime factor is 5, so add 6
21
Not prime, highest prime factor is 7, so add 8
29
Prime, stop

If you start with the number 38, which prime number do you eventually end up at?

249.05 – Descend the Ladder – Elliott Line

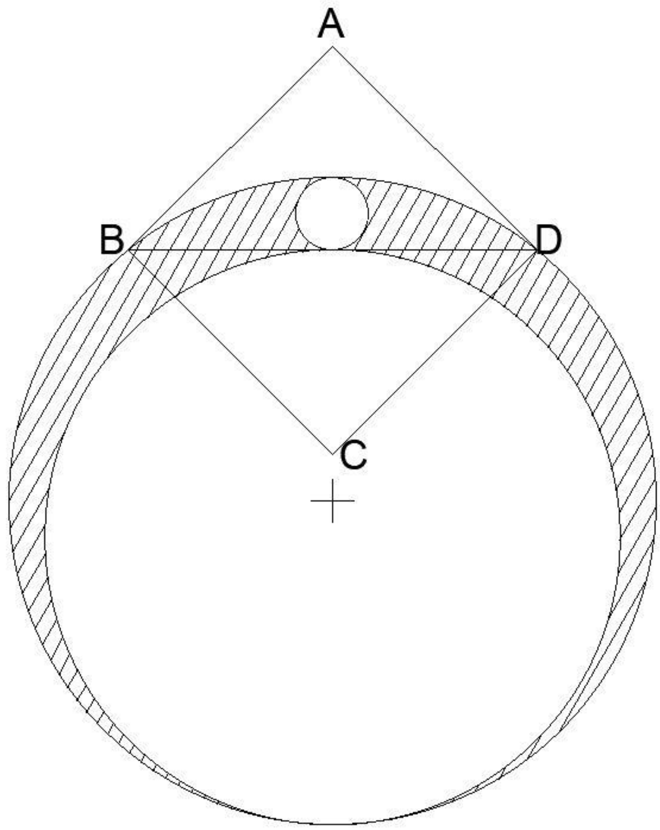
This puzzle is based on a similar algorithm to the last puzzle, but this time the sequence descends from any composite number, and stops when it reaches a number that is not composite (so, either a prime number or the number 1).

At each stage find the largest prime factor of your number and SUBTRACT this plus one from your number to get the next number, for instance: 24 -> 20 -> 14 -> 6 -> 2

There are many, possibly infinitely many, starting numbers that terminate at either 1 or 2, whereas there are no starting numbers that reach 3 or 7. 5 however is a more interesting case, there is a relatively small set of numbers that lead to 5. Can you find them all?

249.06 – Diamond Ring – Elliott Line

ABCD is a square with side length of 2, rotated through 45 degrees so that the diagonal BD becomes horizontal. A circle of unknown radius is drawn through B and D as shown. Within the large circle two further circles are drawn, respectively above and below BD, and the maximum size they can be whilst staying within the large circle.
The region that lies within the large circle but outside of the two smaller circles is shaded. What is the area of the shaded region?



249.07 – Digit Sums – Elliott Line

The longest streak of consecutive numbers, NONE of whose digit sums is a multiple of 7, is 12 in a row. For instance:
994, digit sum = 9+9+4 = 22, remainder after division by 7 = 1
995, digit sum = 9+9+5 = 23, remainder after division by 7 = 2
996, digit sum = 24, remainder after division by 7 = 3
997, digit sum = 25, remainder after division by 7 = 4
998, digit sum = 26, remainder after division by 7 = 5
999, digit sum = 27, remainder after division by 7 = 6
1000, digit sum = 1, remainder after division by 7 = 1
1001, digit sum = 2, remainder after division by 7 = 2
1002, digit sum = 3, remainder after division by 7 = 3
1003, digit sum = 4, remainder after division by 7 = 4
1004, digit sum = 5, remainder after division by 7 = 5
1005, digit sum = 6, remainder after division by 7 = 6

By the same notion, the length of the longest streak of consecutive numbers, NONE of whose digital sums is a multiple of 13, happens to be an exact multiple of 13 itself.
How long is the streak, and can you find an example?

249.08 – Power Sum Palaver – Elliott Line

w + x + y + z = 2
w^2 + x^2 + y^2 + z^2 = 2
w^3 + x^3 + y^3 + z^3 = -4
w^4 + x^4 + y^4 + z^4 = -6

What is the value of: w^7 + x^7 + y^7 + z^7 ?

249.09 – Power Tower – Elliott Line

What are the last five digits of the following number?

$$5^{7^{5^{7^{5}}}}$$

To be clear, in the absence of brackets, a power tower in calculated from the top, so that 3^3^3 = 3^27, not 27^3.

249.10 – Flowers Alphacipher - Brooks Rimes

BONSIA	64
COLEUS	62
DAISES	49
FUCHSIA	98
GERANIUMS	100
GLADIOLI	67
IRIS	7
JUNIPER	78
KOHLRABI	69
LILY	20
MARIGOLDS	80
PEONIES	51
POPPY	39
ROSES	17
TULIPS	65
VIOLET	49
XYLOSMAS	74
YARROW	42
ZINNIAS	91

A		N	
B		O	
C		P	
D		Q	
E		R	
F		S	
G		T	
H		U	
I		V	
J		W	
K		X	
L		Y	
M		Z	

The numbers 1 to 26 have been allocated randomly to the letters of the alphabet.
The letter values of the words have been added together to give the word values.
For example, ROSES might comprise R=4, O=2, S=5 and E=1, or any other combination of numbers totaling 17.
What is the value of F?

249.11 – Highest Prime Factors - Elliott Line

What is the lowest number n such that if a is the highest prime factor of n, then b (not equal to a) is the highest prime factor of (n-a), and c (not equal to a or b) is the highest prime factor of (n-a-b)?

For instance, a higher example is 700. The highest prime factor of 700 is 7, the highest prime factor of 693 is 11, and the highest prime factor of 682 is 31.

249.12 – Missing Digits - Elliott Line

What are the missing digits in this product of fractions?

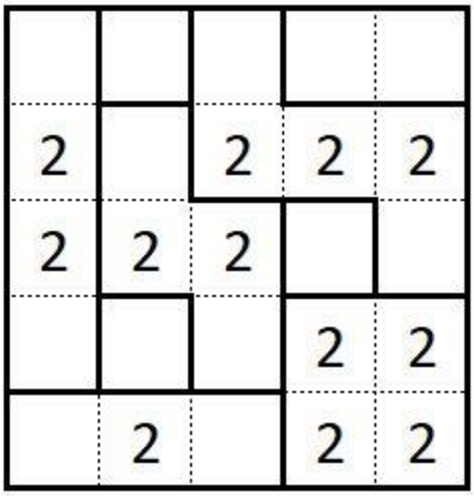
$$2\frac{4}{?} \times 3\frac{?}{9} \times ?\frac{1}{4} = 10$$

249.13 – Galaxy Quest – Elliott Line

Here's a new type of logic puzzle I've invented, inspired by a couple of existing puzzle types (Galaxy and Palisades).
Your task is to subdivide the grid into regions, such that every region has 180-degree rotation symmetry. (They might also have 90-degree symmetry and/or reflective symmetry, but this is not a necessary condition).
Every square that in the solved grid is bordered by 2 or fewer lines, is denoted by the number of lines bordering it. Any square that will have 3 or 4 lines bordering it, is left blank.
The only other rule is that you can't have two single-square regions next to each other, hence why in the example the region in the top right must be a pair.

Example:

2		2	2	2
2	2	2		
			2	2
	2		2	2



Puzzle:

			2	2			2	2
2	2	1	1	2		2	1	1
2		2	1	1	2	2	1	1
	2			2	2		2	2
			2	2	2	1	1	2
	2	2	2		2	1	1	2

249.14 – Rubik's Cube – Elliott Line

As I mentioned briefly in the last issue, just in the past year I have learned how to solve the 3x3 Rubik's cube, as well as a few other variants. I don't want to delve too deeply into methods of solving as many such tutorials are available online. A really good one for the absolute beginner is: https://youtu.be/R-R0KrXvWbc
What I intend to do instead is to talk about what it is that makes the Rubik's cube such a great puzzle.

Some puzzles just have a solution, and once you learn how to solve it, it can get quite boring and same-y.
The Rubik's cube is very different, as we shall see. I remember the board game Othello was for many years marketed with the slogan 'a minute to learn, a lifetime to master'. This equally applies to the Rubik's cube, (the second part anyway).

If you were to follow the tutorial I linked above, and practice that method until you are fluent, you might be able to routinely solve the cube in 1 ½ to 2 minutes. And yet the world record is less than 4 seconds! Surely the world champions and record breakers are using a completely different method? Well, yes and no.
While there are a few different general methods (Roux, ZZ, Petrus) it is generally true that both beginners and champions use the same overall scheme, known as CFOP. CFOP is an abbreviation for the four main steps in the solution (three of which are also abbreviations, confusingly), which I will outline for you now.

Before I start I'll briefly introduce the different types of pieces. It's fairly self-explanatory: each of the six faces has a centre piece, which doesn't move but merely rotates. Because of this, the red centre is always opposite orange, green opposite blue, and yellow opposite white. Edge pieces have two colours and are located next to the centre pieces. There are twelve of these. And each of the eight corner pieces have three colours and are located, well, on the corners.

The C of CFOP stands for Cross. One usually starts with the white cross by convention, but there's no particular reason why you can't start with a different colour, and the best solvers are colour-neutral, meaning they can choose whichever side to form the cross on, based on the scramble. But assuming it IS the white cross, this step involves bringing the four white edge pieces to the white centre. However they need to be in the correct order such that when you line up the red centre with the red/white edge, the green, orange and blue centres also line up with their white edge piece. This step is done fairly intuitively, in contrast to the later steps, which are based on set sequences called 'algorithms'.

F stands for F2L, which is short for First 2 Layers. This step might typically take up about half the time of the overall solve. By the end of this step, the entire white face and two thirds of the red, green, orange and blue faces will be complete, and the only pieces in the wrong place or wrong orientation will be those containing yellow.
This is generally done with the white face downwards. There is a specific difference between how a beginner and an expert will approach this step. A beginner will first place the four white corner pieces, and then place the four edge pieces (those that don't contain white or yellow). This involves repeatedly applying simple algorithms.
An expert will look for a matching corner-edge pair (say, red-white-blue corner and red-blue edge), connect them together and then drop them into the correct 'slot' in one move.

O stands for OLL or Orient Last Layer. This involves making sure all of the yellow faces are facing upwards (but not worrying about whether they are in the right places. For a beginner this involves one algorithm to form a yellow cross, then another to make sure the corners are facing up too. For an expert there are 57 different cases to learn algorithms for to complete this in one step, saving vital seconds. Since this is such a big step a useful halfway house is to learn 2-look OLL. In this method you can always form a yellow cross in one algorithm (three possible cases), and then complete the corners in a second algorithm (seven possible cases).

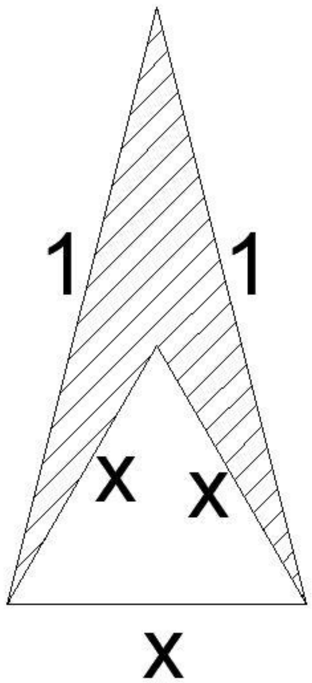
Finally, P stands for PLL or Permute Last Layer. This involves moving the yellow pieces around to complete the solve. For a beginner there are a couple of different algorithms you need: one which cycles the position of three of the edge pieces, and one which cycles the position of three of the corner pieces. Repeatedly applying these will eventually solve any case. An expert will need to learn algorithms for 21 different cases. Again, a useful stepping-stone is 2-look PLL, where you learn the two cases that just change the corners and the four cases that just change the edges, so that you can always solve PLL in two sequences.
Some methods for complete beginners switch back and forth between OLL and PLL stages, for instance: first orient the yellow edges, then permute the yellow edges, then permute the yellow corners, then finally orient the yellow corners.

My own journey has been just a few months from originally learning to solve the cube using the method with the mixed OLL and PLL stage, which by itself is immensely satisfying and completely worthwhile. But then I gained even more satisfaction and pride by learning to insert F2L pairs, and learning 2-look PLL. My personal best time is currently 38 seconds, much improved on the 2 minutes of my early solves. I’m also happy in the knowledge that, if I ever want to, with a bit of dedication I can learn the individual OLL and PLL cases one by one, I could solve the cube in under 30 seconds.

This is just a very brief overview of the most common method of solving, but I hope it gives you some flavour of how learning how to solve the cube is just the very first rung on a very high ladder, and how while it most certainly takes longer than a minute to learn, it could take a lifetime to master.

249.15 – Shaded Area – Elliott Line

In this figure, the shaded area is in between an isosceles triangle with sides 1, 1 and x, and an equilateral triangle with sides x.
If x is close to zero, the isosceles triangle becomes very thin and the shaded area in very small.
If x is close to 1, there is very little difference between the isosceles triangle and the equilateral triangle, and again the shaded area is very small.
In between, the shaded area is at a maximum. For what value of x is this the case?



Usually I like to engineer these puzzles so that the solution is a whole number, however this is not possible in this case, but the exact solution can be expressed quite concisely. But a numerical solution is also acceptable to, let's say, four decimal places.

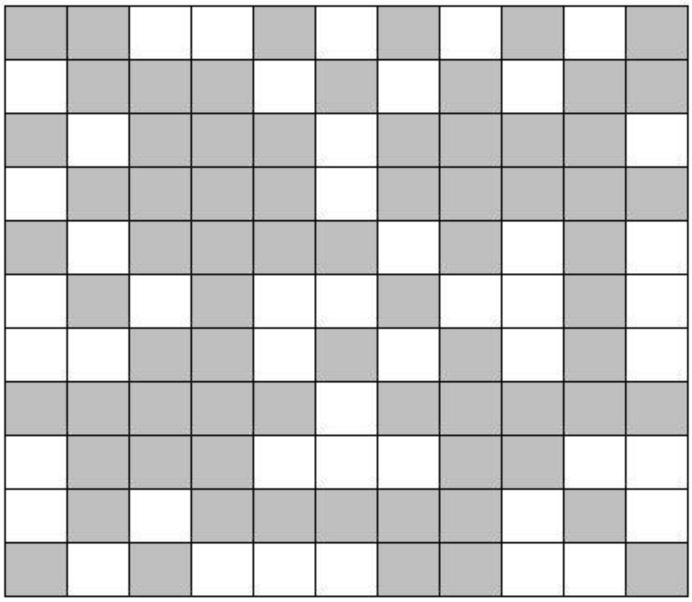
249.16 – Three Fences – Elliott Line

I have three straight lengths of fencing, measuring 25m, 33m and 39m, and a long straight brick wall. Using the wall and the three fences, what is the greatest area I can enclose?

249.17 – Shadowbox – Elliott Line

Here is a puzzle you might not have met before, as it's one of my own invention. A cross between a Fill-In and a Skeleton which I call 'Shadowbox'.

Place all the listed words into the grid, crossword style, such that every white square contains a consonant, and every grey square either contains a vowel (A, E, I, O, U), or becomes a black square. The pattern of black squares in the grid is fully symmetrical.



AHA GYM TANGENT
ATE ICE TRAGEDY
BYE MOO
CUE ODD
DEN OWE ACROBATIC
EAT OWN ALIGNMENT
EEL SAG DOWNLOADS
EGO STY SUPERBOWL
END TEN
ERA USE
ETA WOE
GUN YAM

If you enjoy this, I wrote an entire book of them, available online:
https://www.amazon.co.uk/Shadowbox-Logical-Crossword-Puzzles-Elliott/dp/1447861965

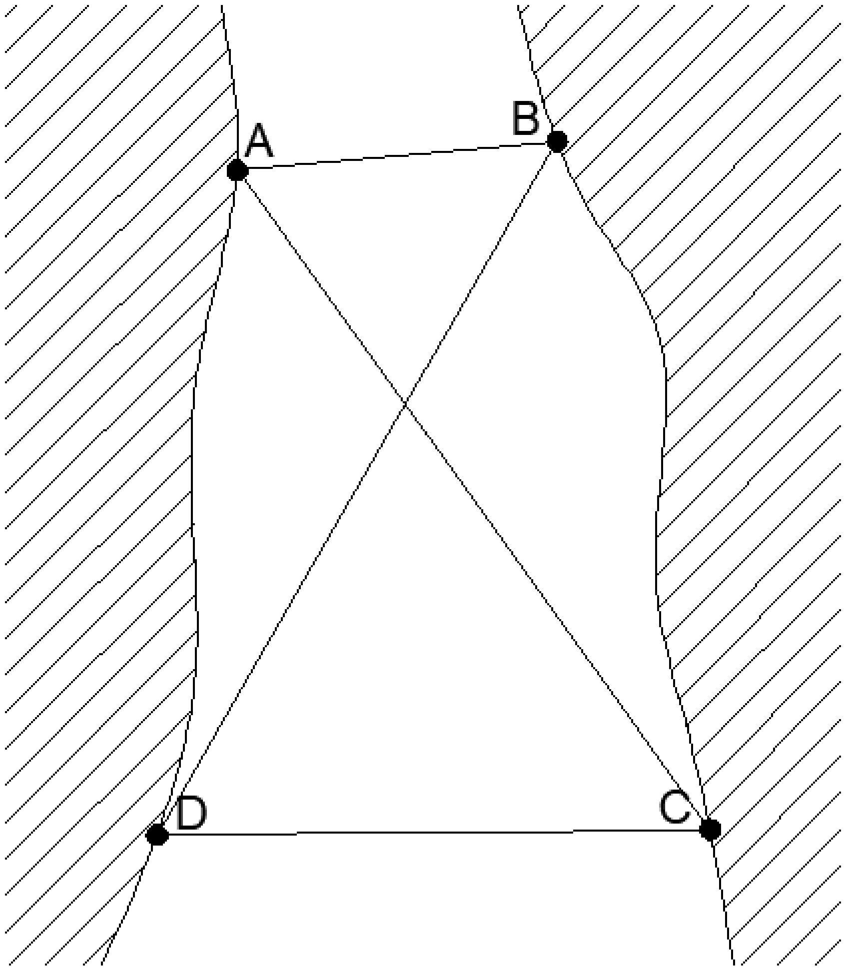
249.18 – Verse – Rosemary Hodgson

A short verse from which spacings have been removed and vowels have been extracted and placed, in order, on the right. Re-insert vowels and word spacings to recreate the verse.

FTDYYPLNTDHP	IOAOUAEOE
NNYHPLSSHRT;	IAOEEEA
FSMN'SBRDN'SLGHTR	IOEOEUEIE
BCSYPLYDYRPT;	EAUEOUAEOUA
FYCSDLGH	IOUAUEAAU
THTCHSDSMTRSWY;	AAEOEEAAA
FTNGHTYRNMSNMD	IOIOUAEIAE
WHNSMNKNLSTPRY;	EOEOEEEOA
THNYRDYHSBNWLLSPNT	EOUAAEEEE

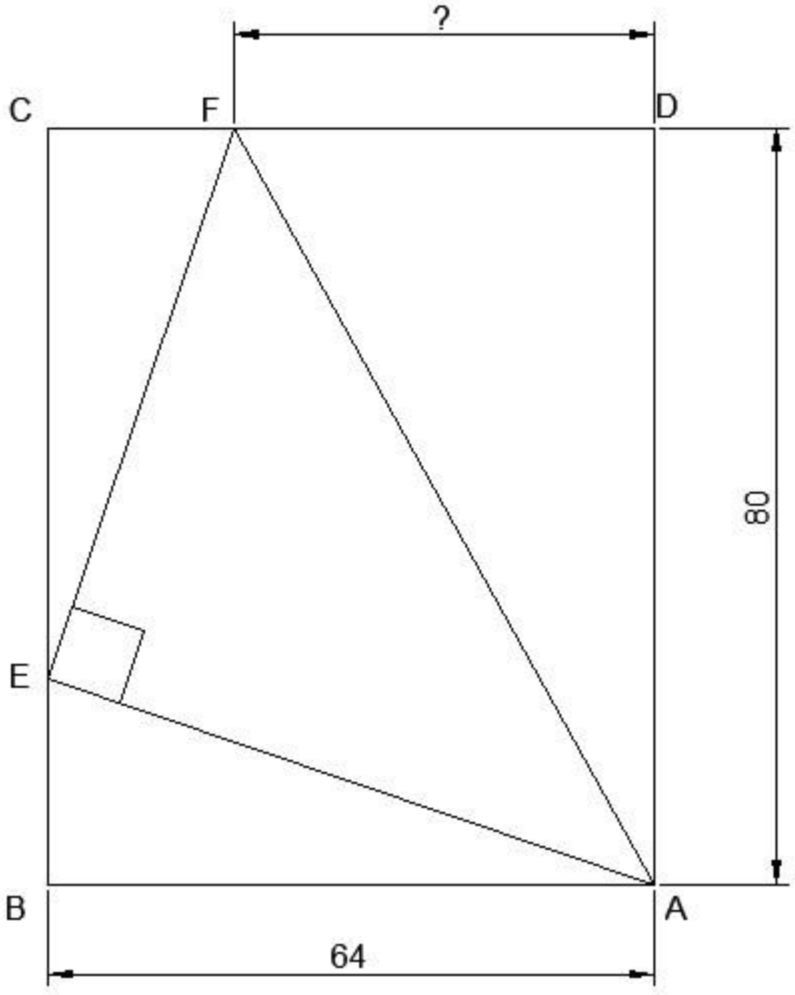
249.19 – Ships that Crash in the Night – Elliott Line

There are two ships, a fast one and a slower one, each travel consistently at their own fixed speed.
If the fast ship travels in a straight line from Duxmouth to Boxcote and the slow ship travels from Axton to Caxcombe, and they each set off at the same time, they will collide where the routes cross.
Similarly, if the fast ship travels in a straight line from Caxcombe to Axton and the slow ship travels from Boxcote to Duxmouth, and they each set off at the same time, they will again collide where the routes cross.
If the fast ship travels from Caxcombe to Duxmouth and the slow ship from Axton to Boxcote, and they set off at the same time, clearly they won't clash, as the routes don't cross, but which ship will reach its destination first?



249.20 – Triangle and Rectangle – Elliott Line

A rectangle ABCD has width 64 and height 80. A right-angled triangle AEF is drawn as shown. What is the minimum that DF could be?



249.21 – Word Search – Graham Holmes

What are these common English words?

1. Length 10, contains the letters G, J, K, W, and Y;
2. Length 11, contains two of each of four different consonants, and two Es;
3. Length 12, contains three of each of three alphabetically consecutive letters;
4. Length 13, contains five identical vowels, each separated by a pair of consonants;
5. Length 14, contains six vowels, all of which are the same letter;
6. Length 15, contains each vowel once only, in reverse alphabetical order.

249.22 – Wolf Tooth – Elliott Line

A Wolf Tooth cube is a strange and interesting puzzle. It is almost like solving two puzzles at once. In essence it is a cube and an octahedron intersected. Each of the six cube faces has one of the octahedron vertices in the centre, a square based pyramid with four different colours on it. Each of the eight octahedron faces has one of the cube vertices in the centre, a triangular based pyramid with three different colours on it.

The arrangement of colours on the cube part are as follows:
Red is opposite Orange
White is opposite Yellow
Blue is opposite Green
Red White and Blue appear clockwise on their shared vertex

The arrangement of colours on the octahedron part are as follows:
Red is opposite White
Yellow is opposite Silver
Purple is opposite Blue
Orange is opposite Green
Red Yellow Purple and Orange appear clockwise around their shared vertex

It is possible to orient the octahedron through the cube such that none of the same colours on the cube and octahedron are in contact?
If so what four colours appear on the octahedron vertex in the middle of the green cube face?

To illustrate the objective of the puzzle, in the cube above the white cube face and the white octahedron face are not in contact, whereas the green cube face and the green octahedron face are in contact, which is not permitted within this puzzle.
(For the purposes of this puzzle I have changed the order of the colours on the octahedron part from the colouring on an actual Wolf Tooth cube, shown here).

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249.02 – Alien Number System – Elliott Line

13041, which in our number system is 3x3x3x3x7x23, in the special number system can either be 21x621 or 81x161, all four of which are prime in the special system (but none prime in real life).

249.03 – Always True? – Elliott Line

Yes, it's always true.

If we call our first number a^2+b^2 and our second number c^2+d^2, since we are told that each is the sum of different squares we can, without loss of generality, specify that a>b and c>d. It is possible for a=c or for b=d, but not both at the same time, since we are told the two sums of squares are different.

If we multiply out (a^2+b^2)(c^2+d^2), we get (ac)^2+(ad)^2+(bc)^2+(bd)^2, which is the sum of four squares, not what we are after. Our approach will be to call the target product (e^2+f^2) and to express e and f in terms of a,b,c,d and demonstrate that (e^2+f^2) is equivalent to (ac)^2+(ad)^2+(bc)^2+(bd)^2.

This can be achieved by letting e=ad+bc and f=ac-bd

e^2+f^2 = (ad)^2+2abcd+(bc)^2+(ac)^2-2abcd+(bd)^2

Clearly the abcd terms cancel out, leaving only the terms we were hoping for.

However, this isn't quite enough to satisfy the question, since e^2 and f^2 need to be different, and our expressions do not guarantee that. For instance if, as in the original example, a=2, b=1, c=3, d=1, both e and f are equal to 5.

Instead let us call g=ac+bd and h=ad-bc (whereas before f was guaranteed to be positive, h now could be negative, but it doesn't really matter, as h^2, which is what we are really interested in, will be positive regardless).

Now g^2+h^2 = (ac)^2+2abcd+(bd)^2+(ad)^2-2abcd+(bc)^2

Again the abcd terms cancel and leave just what we want. But now, since a>b and c>d, g^2 will always be bigger than h^2, meaning we always have two different squares summing to the target value. This method might yield h^2 = 0, which is technically a square number, but if we would prefer non-zero square numbers we can revert to the e and f equations, knowing that if h = 0, e cannot be equal to f. I have a nice proof of this, but I won't go into it now.

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249.04 – Climb the Ladder – Elliott Line

38 +19+1
58 +29+1
88 +11+1
100 +5+1
106 +53+1
160 +5+1
(It's about this point you start to think you'll never hit an odd number, let alone a prime number)
166 +83+1
250 +5+1
256 +2+1
(but by landing on a power of 2 we now change parity)
259 +37+1
297 +11+1
309 +103+1
413 +59+1
473 +43+1
517 +47+1
565 +113+1
679 +97+1
777 +37+1
815 +163+1
979 +89+1
1069, which is prime!

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249.05 – Descend the Ladder – Elliott Line

For a number 'n' to directly reach 5, a number 'm' must exist that is the highest prime factor of n+m+1. If n+m+1 is divisible by m, which is a necessary (but not sufficient) condition for this, then n+1 must also be divisible by m. So we can find all possible candidates for numbers that can directly lead to 5 by looking at the prime factors of n+1. These are 2 and 3.
Considering m=2, our n+m+1 is 8, and 2 is indeed the highest prime factor. So 8 leads to 5.
Considering m=3, our n+m+1 is 9, and 3 is indeed the highest prime factor. So 9 leads to 5.
Now let's consider n=8, the only prime factor of n+1 is 3, so 12 is a possible candidate we need to check. The highest prime factor of 12 is 3, so that works.
Next, n=9 gives us the candidates 12 again, and 15. 15 works as its highest prime factor is 5.
Consider n=12, the only prime factor of n+1 is 13, giving us 26, which works.
For n=15, m=2, the candidate of 18 comes up, however the highest prime factor of 18 is 3, so this doesn't work and proves a dead end.
Finally, n=26, m=3, we need to check to see if the highest prime factor of 30 is 3. It isn't, so this is another dead end.
So the full list of composite numbers that reach 5 is: 8, 9, 12, 15 and 26.

249.06 – Diamond Ring – Elliott Line

If we draw a vertical line through the middle of the figure it should be clear that the diameter of the large circle is the sum of the diameters of the other two circles. It should also be clear that this vertical line will pass through the midpoint of BD. Since we are told ABCD has side length of 2, BD has length sqrt(8) and each half of BD has length sqrt(2).
Using the intersecting chords theorem, we know that the product of the two smaller diameters must be the same as the product of the two halves of BD, namely 2.
If we call the RADIUS of the smallest circle x and the radius of the other inner circle y, then we know that 2x*2y =2, and so xy=1/2.
The outer circle has radius z = x+y
To find the shaded region we simply need to find the area of the outer circle and subtract the areas of the other two circles.
Shaded region S = (pi)z^2 – (pi)x^2 – (pi)y^2.
S/pi = z^2 – x^2 – y^2
= (x+y)^2 – x^2 – y^2
= x^2 + 2xy + y^2 – x^2 – y^2
= 2xy
But we know xy=1/2, and so S/pi = 1.
Therefore the shaded region S = pi, and is not dependent upon the size of the circles.

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249.07 – Digit Sums – Elliott Line

The longest possible streak is 78.
The first such streak starts with 9,999,999,961 and continues until 10,000,000,038.

249.08 – Power Sum Palaver – Elliott Line

The answer is 16
w^5 + x^5 + y^5 + z^5 = -8
w^6 + x^6 + y^6 + z^6 = 2
w^7 + x^7 + y^7 + z^7 = 16

The values of w,x,y,z in some order are 1, -1, 1+i, 1-i.

249.09 – Power Tower – Elliott Line

If you try to calculate the number you won't get very far. 5^7^5 is already way too large for a calculator, at over 11000 digits. The number of digits in the entire number is unimaginably large. But of course we only need the final five.
If we look at the pattern of the final five digits of 5^n, (also known as the remainder of 5^n when divided by 100000), we see a nicely repeating pattern. Once the numbers are big enough, the final five digits cycle around just eight options.
So to find the last five digits of 5^(very large number), we only need to know the remainder of the (very large number) when divided by 8.
But the (very large number) we are concerned with is equal to 7^(quite large number), and the remainder of 7^n when divided by 8 alternates between just two numbers: 1 when (quite large number) is even and 7 when (quite large number) is odd.
But the (quite large number) is equal to 5^(large number). 5 to the power of any whole number will be odd, since 5 is odd. Therefore 7^(quite large number) gives a remainder of 7 when divided by 8.
Therefore the final five digits of 5^(very large number) are 78125.

249.10 – Flowers Alphacipher - Brooks Rimes

F is worth 19. The full solution is as follows:
A14 B16 C18 D24 E4 F19 G10 H17 I1 J11 K8 L6 M15 N25 O5 P9 Q23 R2 S3 T20 U26 V13 W12 X21 Y7 Z22

249.11 – Highest Prime Factors – Elliott Line

ANSWER = 128
The highest prime factor of 128 is a=2,
The highest prime factor of 126 is b=7,
The highest prime factor of 119 is c=17.

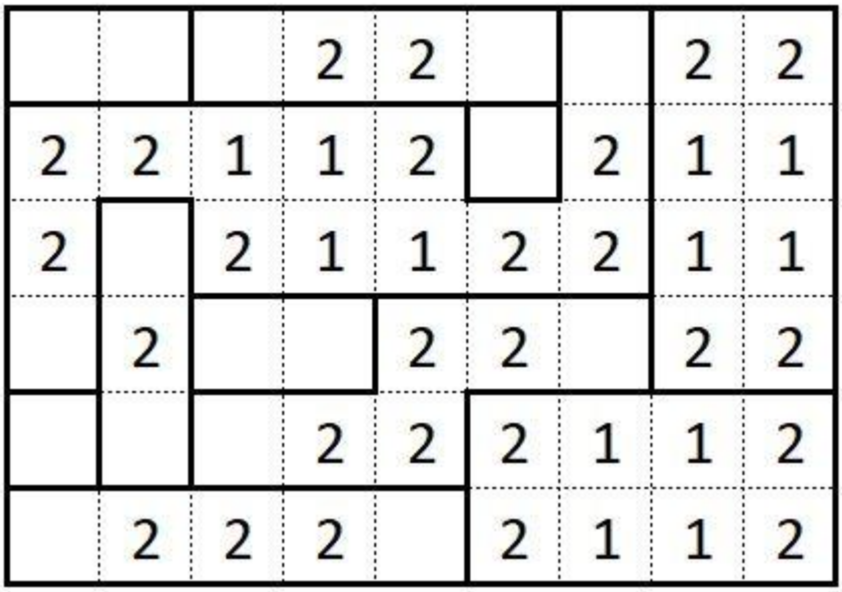
249.12 – Missing Digits – Elliott Line

7, 1 and 1.
2 4/7 x 3 1/9 x 1 1/4 = 10

~~~ SOLUTIONS ~~~ SOLUTIONS ~~~ SOLUTIONS ~~~

249.13 – Galaxy Quest – Elliott Line

Solution:



249.15 – Shaded Area – Elliott Line

If you draw a vertical line between the apexes of the two triangles you will exactly divide the shaded area in two. Each half will have a side of 1 and another of x, and also a fixed angle (opposite the side 1) of 150 degrees. In order to maximise the area of this triangle, we must maximise the distance between the vertex with angle 150, and the side of length 1. This will be achieved when the triangle is isosceles, such that the distance between the two apexes is also x.

Firstly we need an exact value for the cosine of 150 degrees. You could of course look it up but I’ll quickly derive it.

Cut an equilateral triangle of side length 2, in half and you’ll have a right-angled triangle with angles 30, 60 and 90, with a hypotenuse of 2 and a side of length 1 opposite the 30 angle. From Pythagoras the third side is the square root of 2^2 – 1^2, so is the square root of 3. Therefore the cosine of 30 is sqrt(3)/2. The cosine function has rotational symmetry around the 90 degree point, which means that the sum of cos(a) and cos(180-a) will be 0 for all a, therefore the cosine of 150 is simply the negative of the cosine of 30. Cosine 150 is therefore -sqrt(3)/2.

Since cosine of 150 degrees is -sqrt(3)/2, using the cosine rule it’s possible to calculate x to be sqrt(2-sqrt(3)), which is approximately 0.5176.
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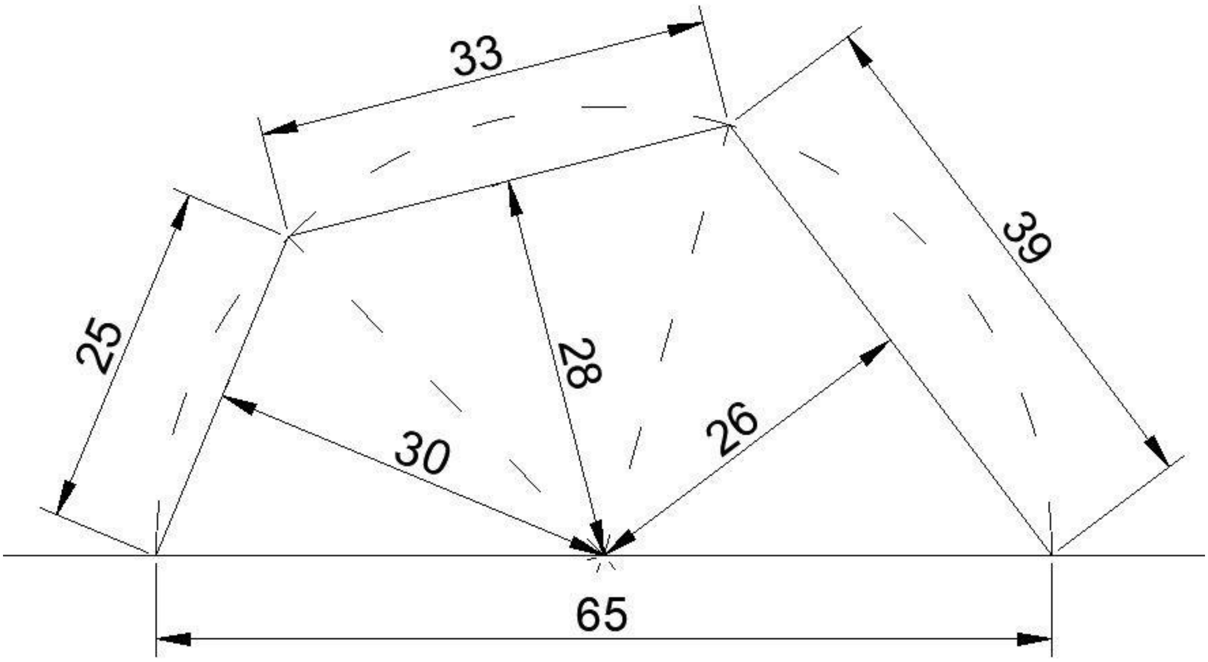
249.16 – Three Fences – Elliott Line

The greatest area bounded by four straight edges occurs when those edges form a cyclic quadrilateral (all four vertices lie on a circle). Given that we can choose the length of wall that we use, it makes send for that to be the diameter of the circle, with length 2r. Then the three fences each form isosceles triangles with r as the common lengths and 25, 33 and 39 as the 'bases'. It turns out that for this to happen, r=32.5, and the three triangles have 'heights' of 30, 28 and 26 respectively. The area is therefore 1344 square metres.



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249.17 – Shadowbox – Elliott Line

	A	C	R	O	B	A	T	I	C	
S		U		W		H		C		A
U	S	E		E	T	A		E	E	L
P					R					I
E	R	A		E	A	T		S	A	G
R		T	A	N	G	E	N	T		N
B	Y	E		D	E	N		Y	A	M
O					D					E
W	O	E		G	Y	M		O	W	N
L		G		U		O		D		T
	D	O	W	N	L	O	A	D	S	

249.18 – Verse – Rosemary Hodgson

If today you planted hope in any hopeless heart;
If someone's burden's lighter, because you played your part;
If you caused a laugh which chased some tears away;
If tonight your name is named when someone kneels to pray;
Then your day has been well spent.

249.19 – Ships that Crash in the Night – Elliott Line

Because of similar triangles, the two ships will each reach their destination at the same time.

249.20 – Triangle and Rectangle – Elliott Line

The minimum length of DF is 39.

249.21 – Word Search – Graham Holmes

1. JAYWALKING
2. CONDESCENDS
3. HIGHLIGHTING
4. EFFERVESCENCE
5. INDIVISIBILITY
6. UNCOMPLIMENTARY
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249.22 – Wolf Tooth – Elliott Line

Below is a diagram showing the necessary arrangement of the colours. The cube colours are in the squares, and the octahedron colours are in the hexagons.

The colours on the octahedron vertex in the middle of the green cube face are: Red, Orange, Silver, Blue.