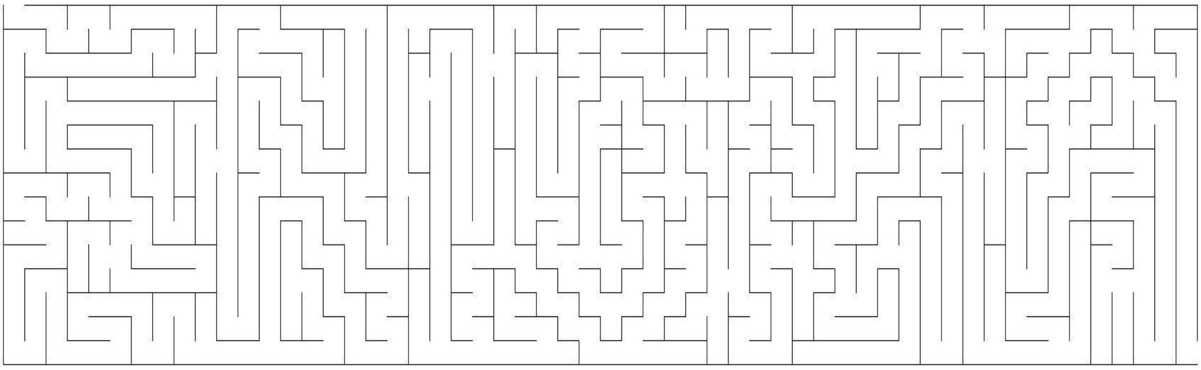
Enigma:
The Newsletter of PuzzleSIG



ENIGMA

250

'Mathematical problems, or puzzles, are important to real mathematics, just as fables, stories, and anecdotes are important to the young in understanding real life'.

Terry Tao

November 2021

Electronic Version of this Newsletter
Email enigma.mensa@yahoo.co.uk and I'll send you a copy

About Enigma
Enigma is the newsletter of Puzzle SIG.
The SIG for anyone interested in puzzles. The scope covers word puzzles, crosswords, logic puzzles, Japanese puzzles, mathematical brain teasers, lateral thinking problems, quizzes and picture quizzes, discussion of physical / mechanical puzzles, computer / internet based puzzles and puzzle games, and puzzle books and publications, and experimentation and innovation of new puzzle forms.

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How to Join
You can join Puzzle SIG by visiting mensa.org.uk/sigs (member login required).

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Welcome to Enigma 250

Hello and welcome to another issue of Enigma.

Apologies for taking so long between issues, I took a break from many different things earlier in the year in order to look after my mental health. I am feeling much better now, and able to get this newsletter out to you in time for the festive season, where if you have a spare moment, you can pit tour wits against the puzzles herein.

There were not many answers sent in for the competition puzzle last time around, but I believe you'll find the new one far more approachable. To be honest I'm more of a runner than a cyclist myself, but 250km is a little bit too far to run!

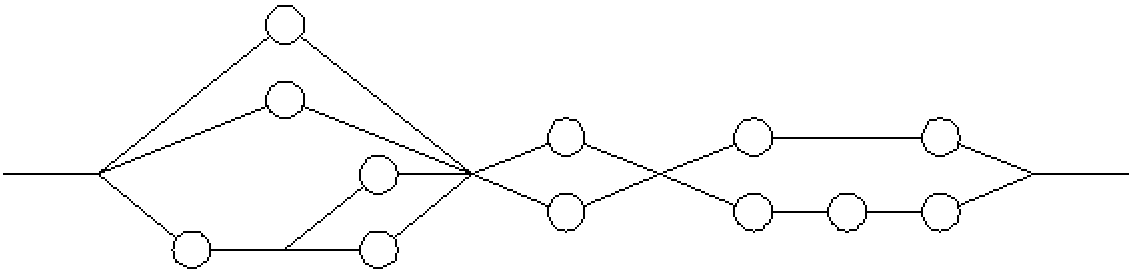
Thanks to Roisin Carters and Goran Grkinic for their contributions.

I should say that I had a few problems with my email account, and it's possible that some emails were lost. If you have sent me an email and your contribution is not here, please accept my apology and please re-send to the usual email address: enigma.mensa@yahoo.co.uk

Please try all of the puzzles, and if you have any feedback, let me know and I'll pass it on to the puzzle's creator.
As ever, if you get stuck and need a hint, drop me a line and I'll be happy to help.
Please keep your puzzles, answers, comments, queries, suggestions, etc coming in.

Happy puzzling
Elliott.

249.01 - COMPETITION: Run with Resistors - Elliott Line



The three sections above total 45, 60 and 144 respectively, summing to 249 ohms.

WELL DONE TO:
Stuart Nelson
Jeff Jowers
Abhilash Unnikrishnan
Christa Ramonat
Karl Billington

250.01 - COMPETITION: Cycling Laps - Elliott Line

There is a cycling race which is 250km long (250 being the number of this issue of Engima), and has markers at every kilometre. The course consists of 9 laps, each 27 7/9 kilometres long. Because of this, the cyclists will first encounter the kilometre markers in a peculiar order, for instance, 2/9 kilometre from the start, they will pass the 28km marker, but they won't have actually travelled 28km until they pass it again on the second lap.

What kilometre markers will the cyclists pass, and in what order, before they first reach the 1km marker?

This is a competition, but not for prizes, only bragging rights. Every correct answer I receive will get an honourable mention in the next issue of Enigma. Send your answer to me at enigma.mensa@yahoo.co.uk .

250.02 – Binary Determination - Elliott Line

Place either a 0 or a 1 in each empty square such that each row and each column will form a correct multiplication of two binary numbers to give the decimal number at the end of the row or column. For instance, if the product is 12, the row might read 011x100 (3x4), or one of a number of other possible combinations. I have provided decimal-to-binary tables to help you. Here are two puzzles, one with binary numbers of up to three digits, and one up to four digits.

			x				=	1
			x				=	18
			x				=	49
x	x	x		x	x	x		
			x				=	12
			x				=	35
			x				=	4
=	=	=		=	=	=		
18	3	10		2	21	42		

1	001
2	010
3	011
4	100
5	101
6	110
7	111

				x					=	15
				x					=	18
				x					=	84
				x					=	60
x	x	x	x		x	x	x	x		
				x					=	60
				x					=	105
				x					=	120
				x					=	44
=	=	=	=		=	=	=	=		
84	154	130	65		30	15	84	32		

1	0001	9	1001
2	0010	10	1010
3	0011	11	1011
4	0100	12	1100
5	0101	13	1101
6	0110	14	1110
7	0111	15	1111
8	1000		

250.03 – Consecutive Missing Letters - Elliott Line

The missing letters in each of the following words are found next to each other in the alphabet. The missing letters might or might not be repeated amongst the missing letters, for instance _ _ _ _ _ G E could be CABBAGE, since A B and C are to be found together in the alphabet.
Complete each of the following words using a set of five alphabetically consecutive letters in each case. Each uses a different set of five letters, but there may be some overlap.

_ L _ M _ _ R _ _

_ _ R _ _ _ _ _ T _ R

_ E _ _ E _ _

250.04 – Base 4 Riddle – Elliott Line

I have taken a riddle, and I have replaced each of the letters with the numbers that denote their position in the alphabet. However, I have used the base 4 number system. Be careful, as some sequences of numbers could lead to several words, for instance 31110 could mean CAT (3,1,110), but could equally mean MAD (31,1,10). Can you decode the riddle and then solve it?

	1 = A	2 = B	3 = C
10 = D	11 = E	12 = F	13 = G
20 = H	21 = I	22 = J	23 = K
30 = L	31 = M	32 = N	33 = O
100 = P	101 = Q	102 = R	103 = S
110 = T	111 = U	112 = V	113 = W
120 = X	121 = Y	122 = Z	

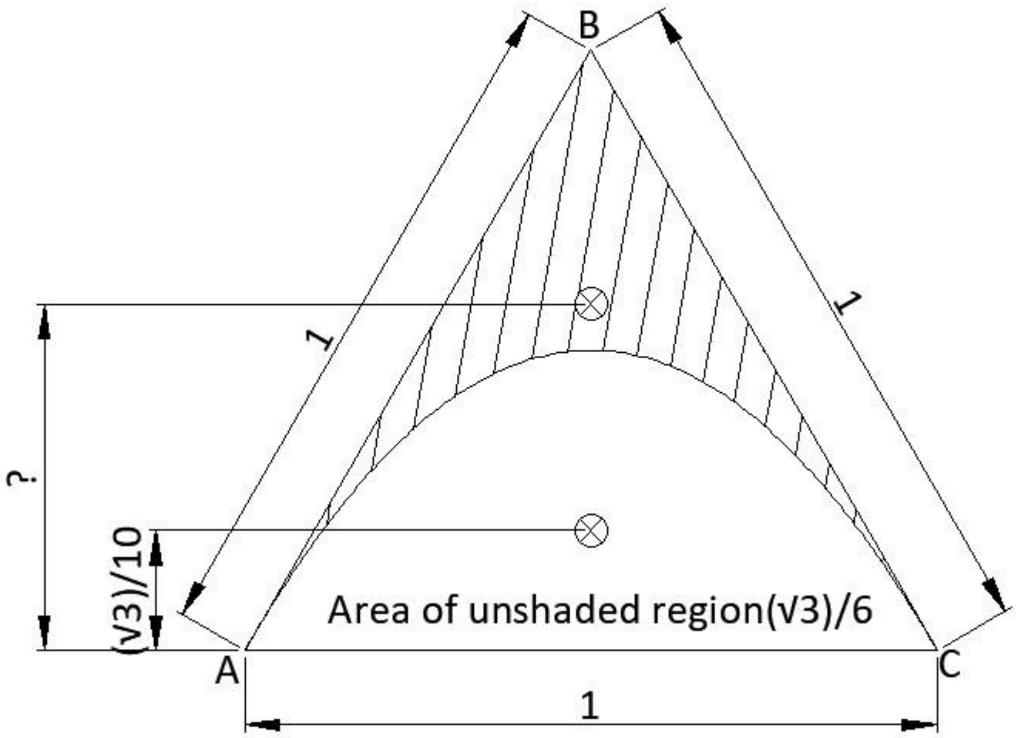
21'31 12331113210 2132 10333323103, 1033110211211103 13210 31211101101132103;

13210 33121101132 2132 1102011 1001113103 3312 1003011211211130

23211101101132103. 113201110 131 21?

250.05 – Envelope Centroid - Elliott Line

ABC is an equilateral triangle. If I tell you that the area of the unshaded region is sqrt(3)/6 and that the centroid (centre of gravity) of the unshaded region is sqrt(3)/10 above AC, how far above AC is the centroid of the shaded region?



250.06 - Even Binary Puzzle - Elliott Line

Consider a process by which you take a binary number with an even number of 1s, (for instance 92 = 1011100 has 4 1s), chop the number into two chunks such that each chunk has the same number of 1s, and each chunk begins with 1 (101=5 and 1100=12). Then multiply these two numbers together to give a number smaller than the one you started with (111100=60). If the new number has an even number of 1s, repeat the process, otherwise stop. (11=3 and 1100=12 gives 100100=36; 100=4 and 100=4 gives 10000=16; stop).

25 and 32 both convert to binary numbers with an odd number of 1s, (11001 and 100000 respectively).

What is the highest number that will eventually terminate in at 25?

As a bonus question, what is the highest number that will eventually terminate at 32?

250.07 – False Conjecture – Elliott Line

The sequence: 1, 21, 321, 4321, etc is very simple to generate. It is the sum of k(10^(k-1)) for all k between 1 and n, with the results of n=1, 2, 3, 4 shown above. It only gets slightly messier when n is in double digits, for instance the 14th number in the sequence is: 14320987654321, and the 28th number is 30987654320987654320987654321.
It was conjectured that no number in this sequence is prime, but that turns out to be false.
Can you find the first counterexample? Just to warn you this cannot be done without a computer.

250.08 - Cryptogram - Roisin Carters

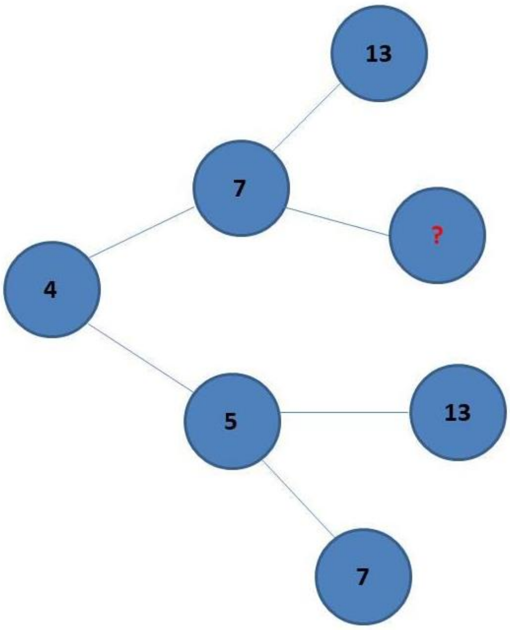
ABC DEF GHI ABC JKLLM NGA ECHA AD LCG OH G PCGKAOQKF JCG RSCCH PDGA. ABCM ADDT LDUC BDHCM GHI JFCHAM DQ UDHCM ESGJJCI KJ OH G QOVC JDKHI HDAC.

General hint: Take a frequency count of each letter and compare these to the most common letters used in English text, considering often-used short words first.

250.09 - Sequence - Goran Grkinic

12, 26, 82, 54, ?

250.10 - Missing Number - Goran Grkinic



250.11 – Inch Centimetre Prime – Elliott Line

I am thinking of a distance, less than 200m, which is an exact whole number of inches and also an exact whole number of centimetres (one inch is precisely 2.54cm).
The number of inches is one more than a prime number.
The number of centimetres is one more than a prime number.
The sum of the number of inches and the number of centimetres is also one more than a prime number.

What is the distance?

250.12 – Maximum Quotient - Elliott Line

Take a five-digit-number and split it at some point between the digits into two shorter numbers (for instance 12345 could be split into 123 and 45), then multiply together the two shorter numbers. Take your original five-digit-number and divide by this product.
In our example this quotient would be 12345 / (123 x 45) = 2.23…
What is the highest this quotient can be, and what is the five-digit-number that achieves it?

Note: each of the two shorter numbers must start with a non-zero digit, so 12003 could split into 1 and 2003, or 1200 and 3, but not 12 x 003.

250.13 – Non-sequitur - Elliott Line

Attempt this question without any electronic assistance.

Given that the square root of 10 is about 3.16,
how many 5- or 6-digit number are there with exactly 5 factors (including 1 and itself)?

250.14 – Number Hunt: p and p-1 - Elliott Line

I have a number, let's call it n. n doesn't have any repeated prime factors. For every prime number p, p divides into n IF AND ONLY IF p-1 also divides into n.
For instance, if 23 divides into n, then 22 does also, and if 59 doesn't divide into n, then neither does 58.
What is the value of n?

250.15 – Number Hunt: p+2 = sp - Elliott Line

I have a number, n, for which it is true that whenever a semiprime is 2 greater than a prime, they are either both factors of n, or neither are. A semiprime is a number with exactly two prime factors, for example 4 (2x2) or 6 (2x3).
So for instance, 33 is a semiprime (3x11) which is 2 greater than a prime (31), and so EITHER 33 and 31 both divide into n, OR neither do.

My number only has one pair of repeated prime factors: a pair of 3s. All its other prime factors are unique.
n is the smallest possible number to satisfy the above rules. What is n?

For an extra challenge, what is the next smallest number that satisfies the rules?

250.16 – Number Hunt: 7 digit number - Elliott Line

Can you find a seven-digit number containing seven different digits, whose prime factors are four two-digit primes containing between them eight different digits?

250.17 – Self-Assembly - Elliott Line

Assemble these two- and three-letter chunks into six surnames of famous people with a specific occupation in common:

ANE	BO	CH	CHA	ER	ETT
HA	ILL	LL	MM	NDE	NDL
NES	RE	RIS	SP	TIE	

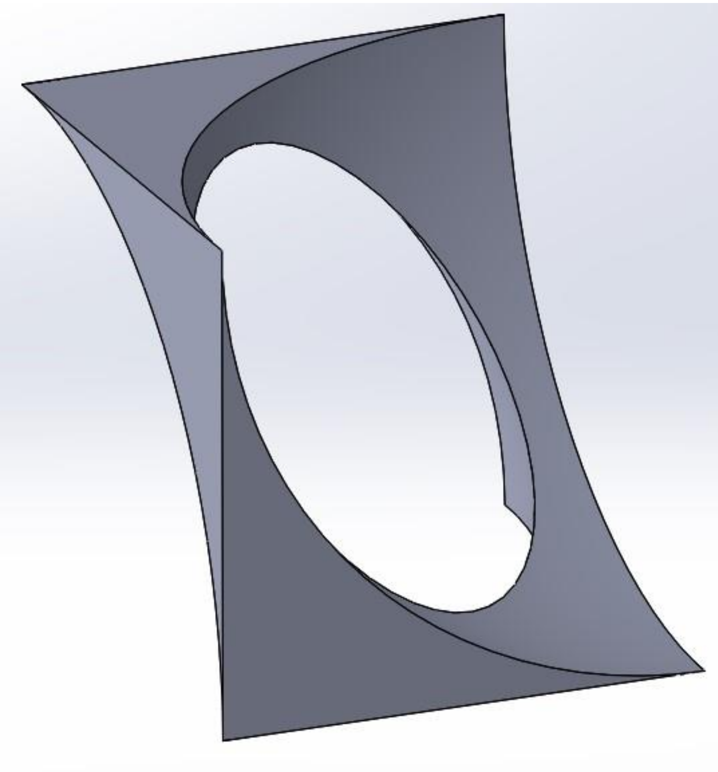
250.18 – Sixth Powers - Elliott Line

All square numbers are either a multiple of 5 or one away from a multiple of 5. 5 is the highest number for which this is true.
For cube numbers, the highest number that every cube number is at most one away from a multiple of, is 9.

What is the highest number for which it is true that every sixth power (1, 64, 729, 4096, etc) is at most one away from?

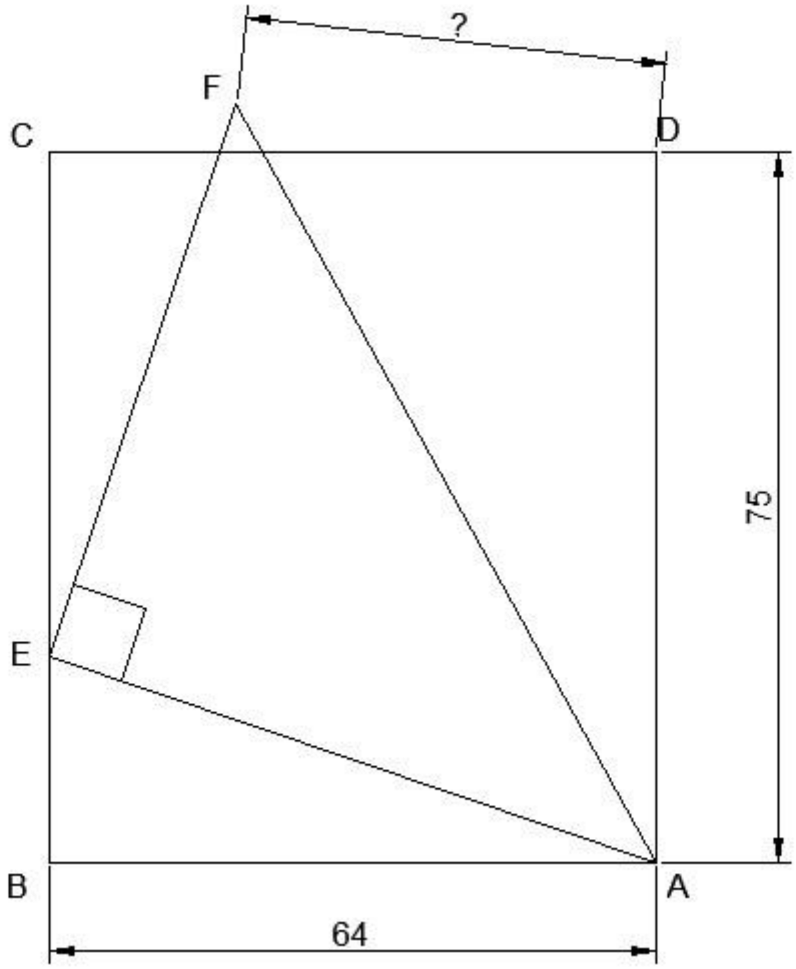
250.19 – Skeleton Cube - Elliott Line

I have taken a unit cube and removed any material that coincides with a unit radius sphere centred on one of the vertices, then I have removed any material that coincides with another unit radius sphere centred on the opposite vertex. This has resulted in a circular hole in the middle of what is left of the cube. What is the diameter of this circular hole?



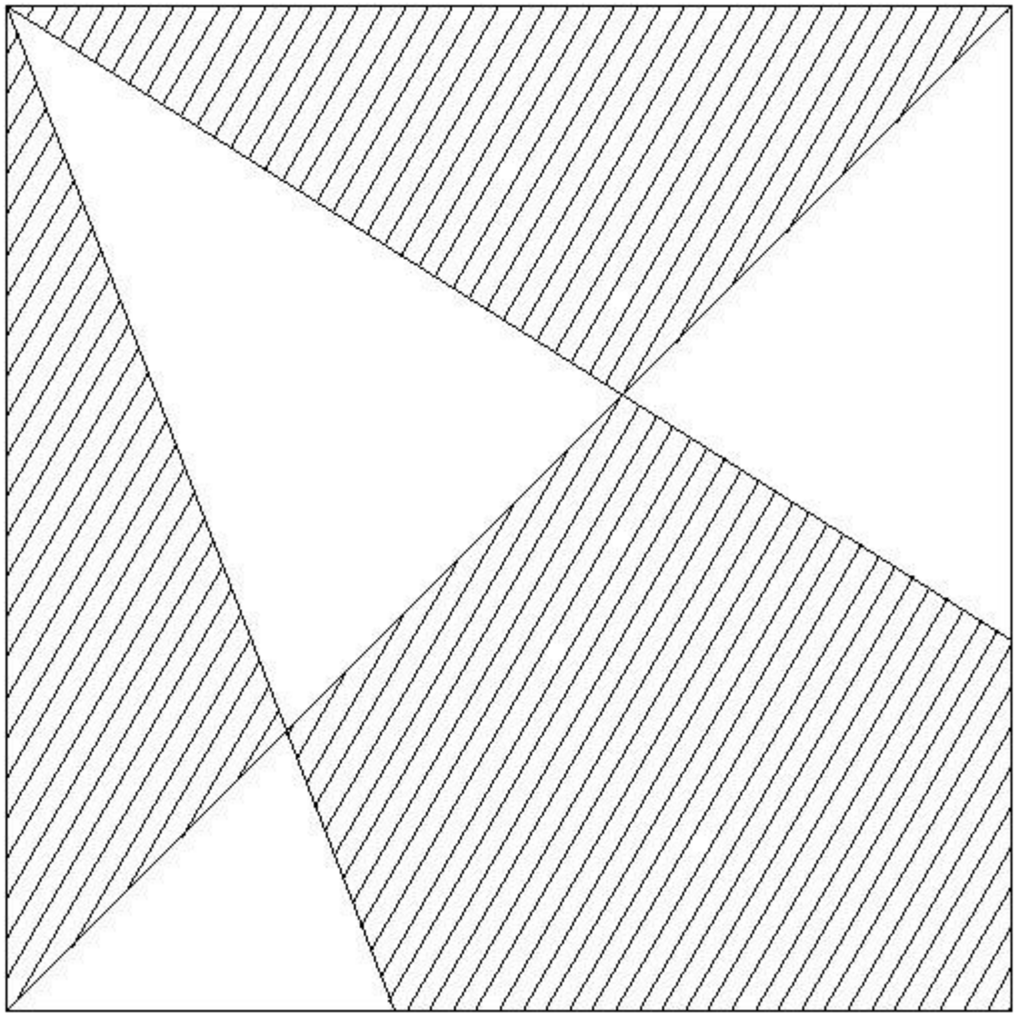
250.20 – Triangle and Rectangle 2 – Elliott Line

This is quite similar to PotW #320 (Enigma 249.20) but differs in a couple of respects. Firstly, the rectangle is a slightly different height. Secondly, the point F no longer needs to lie on CD, but is free to be anywhere, subject to the other conditions (that ABCD is a rectangle, AEF is a right angle, and E lies on BC.
What is the minimum that DF could be?



250.21 – Unit Square - Elliott Line

Three straight lines are drawn in a 1 x 1 square as shown, such that one line extends between opposite corners, and the other two line meet at one of the other corners, with the other end of each line meeting the edge of the square partway along an edge.
What is the maximum possible combined area of the shaded regions?



250.22 - Uprooted - Elliott Line

If (x times the square root of y), plus (y times the square root of x) is 14,
and (x times the square root of x), plus (y times the square root of y) is 22

What is the value of (x plus y)?

SOLUTIONS

250.02 – Binary Determination - Elliott Line

0	0	1	x	0	0	1	=	1
1	1	0	x	0	1	1	=	18
1	1	1	x	1	1	1	=	49
x	x	x		x	x	x		
1	0	0	x	0	1	1	=	12
1	0	1	x	1	1	1	=	35
0	1	0	x	0	1	0	=	4
=	=	=		=	=	=		
18	3	10		2	21	42		

1	1	1	1	x	0	0	0	1	=	15
1	0	0	1	x	0	0	1	0	=	18
0	1	1	0	x	1	1	1	0	=	84
0	1	0	1	x	1	1	0	0	=	60
x	x	x	x		x	x	x	x		
0	1	1	0	x	1	0	1	0	=	60
1	1	1	1	x	0	1	1	1	=	105
1	1	0	0	x	1	0	1	0	=	120
1	0	1	1	x	0	1	0	0	=	44
=	=	=	=		=	=	=	=		
84	154	130	65		30	15	84	32		

250.03 – Consecutive Missing Letters - Elliott Line
CLAMBERED
FIREFIGHTER
REQUEST

250.04 – Base 4 Riddle – Elliott Line
I'm found in socks, scarves and mittens; and often in the paws of playful kittens. What am I?
Wool!

250.05 – Envelope Centroid - Elliott Line
The centroid of a shape is the weighted average of the centroids of its component parts.

$A_1.C_1 + A_2.C_2 = A.C$

Where A_1 is the area of the unshaded region, C_1 is the centroid height of the unshaded region etc.

We require C_2, so making that the subject:

$C_2 = (A.C - A_1.C_1)/A_2$

We can calculate the area and centroid height (A and C) of the overall figure from first principles, as it is simply an equilateral triangle, and we can find A_2 by simply taking A_1 from A.

C_2 = (rt(3)/4 x rt(3)/6 - rt(3)/6 x rt(3)/10) / (rt(3)/12)

C_2 = 3rt(3)/10 =~ 0.5196…

250.06 - Even Binary Puzzle - Elliott Line

The highest number that will terminate at 25 is 123:

123	1111011	111 x 1011	7 x 11
77	1001101	1001 x 101	9 x 5
45	101101	101 x 101	5 x 5
25	11001	stop	

The highest number that will stop at 32 is 965:

965	1111000101	111 x 1000101	7 x 69
483	111100011	111 x 100011	7 x 35
245	11110101	111 x 10101	7 x 21
147	10010011	100100 x 11	36 x 3
108	1101100	110 x 1100	6 x 12
72	1001000	100 x 1000	4 x 8
32	100000	stop	

250.07 – False Conjecture – Elliott Line

The 58^{th} number:
64,320,987,654,320,987,654,320,987,654,320,987,654,320,987,654,320,987,654,321
is the first in the sequence to be prime.

The 40^{th} number:
44,320,987,654,320,987,654,320,987,654,320,987,654,321
is tempting as it doesn't have any prime factors smaller than 78 million, but it is in fact composite.
Several others only have two prime factors.

250.08 - Cryptogram - Roisin Carters

The owl and the pussy cat went to sea in a beautiful pea green boat. They took some honey and plenty of money wrapped up in a five pound note.

250.09 - Sequence - Goran Grkinic

The answer is 74.
1.) Switch the digits
2.) Add the sum of the digits
3.) Add the product of digits

E.g. 12 = 21 + (2+1) + (2x1) = 26
....
54 = 45 + (4+5) + (4x5) = 74!

250.10 - Missing Number - Goran Grkinic

4 is connected with 7 and 5 (7+5=12)
5 is connected with 13 and 7 (13+7=20)
7 is connected with 13 and "X" (13+ "X")

4:12 must be in the same corelation as 5:20 ;

4x4-4=12
5x5-5=20
...so, we have 7x7-7=42
...meaning 13+X=42
X=29!

250.11 – Inch Centimetre Prime – Elliott Line
To be a whole number of centimetres, the number of inches must be a multiple of 50. For the number of centimetres to be even, the number of inches can be further narrowed down to multiples of 100. For both the number of inches and the number of centimetres to avoid being one more than a multiple of 3, the number of inches must be a multiple of 300. This straight away narrows down the task to the point where we can check individual cases.
If the number of inches were 2100, that would convert to 5334cm: 2099, 5333 and 7433 are all prime.
So the distance is 2100 inches or 5334 cm.
This is the only possible distance under 200m as the next set of numbers that fit the criteria are: 9000 inches (22860cm).

250.12 – Maximum Quotient - Elliott Line
11000 / (1 x 1000) = 11

250.13 – Non-sequitur - Elliott Line
Since 5 is a prime number, the only way a number can have exactly five factors is if it is the fourth power of a prime number, in which case the factors will be 1, p, p^2, p^3 and p^4.
Therefore, to answer the question we need to find out how many prime numbers lie between the fourth root of 10^4 and the fourth root of 10^6. The fourth root of 10^4 is clearly 10. The fourth root of 10^6 is 10^(6/4) which is 10^(3/2) or 10 x 10^(1/2), in other words 10 x the square root of 10. The square root of 10 is given in the question as about 3.16, so therefore the fourth root of 10^6 is about 31.6.
So the answer we seek is the number of prime numbers between 10 and 31.6, which is straightforward enough to simply count: 11, 13, 17, 19, 23, 29, 31 which gives the answer of 7.

250.14 – Number Hunt: p and p-1 - Elliott Line
Since p-1 will be even for any odd prime, 2 must be a prime factor of n, and as it is also one less than a prime, so is 3.
Since 2x3 is one less than a prime, 7 is also a prime factor of n.
Of the products of subsets of 2,3,7, only 2x3x7 is one less than a prime, so 43 is also a prime factor of n.
Of the products of subsets of 2,3,7,43, none are one less than a prime, so we can stop there.
n = 2x3x7x43 = 1806.

250.15 – Number Hunt: p+2 = sp - Elliott Line
Since we are told that a pair of 3s are amongst the prime factors of n, the semiprime 9 is a factor, and since 9 is 2 greater than a prime, 7 is also a factor. So another semiprime factor is 21, therefore 19 is a prime factor. Of the semiprime factors involving 19, 133 is 2 greater than a prime number, so 131 is also a prime factor of n. Of the semiprimes involving 131, there are none that are 2 greater than a prime, so we can stop there.
n is therefore 3x3x7x19x131 = 156807.
There could be other factors, for instance if 41 were also a prime factor, the rule would still be satisfied, since 41 in not 2 less than a semiprime, and each of the semiprimes involving 41 would not be 2 greater than a prime, so 156807 is merely the smallest.

For the second challenge, we need to introduce another prime factor, as small as possible.
It cannot be 2, as that would imply 4 divides too, but we know we can't have any more repeat prime factors.
If it was 5, that implies 13 is also a factor, so we'll put that to one side.
If it was 11, 31 would also be a factor.
13 implies 5 is a factor, as we've already seen.
If 17 is a factor, none of the semiprimes formed with it are 2 greater than a prime, and 17 itself is not 2 less than a semiprime, so we can stop there.
Therefore multiplying the previous answer by 17 gives the next smallest: 2665719.

250.16 – Number Hunt: 7 digit number - Elliott Line
5623109 = 23 x 41 x 67 x 89
There are in total 12 ways of combining four two-digit primes with no digits in common, but only one of the twelve products contains no repeating digits.
This is how you can reduce the number of possible arrangements to just twelve (which can then be easily checked to see if the product contains no repeated digits), just with a little logic:
Once we are past single digit primes, all primes must end in 1, 3, 7, or 9. Since we are after four primes with no repeated digits, they must end in these four digits, and therefore they cannot start with those four digits. They also cannot start with a 0 as they wouldn't be two-digit numbers, so the start numbers must be four of the following five: 2,4,5,6,8. Since the primes starting with 2,5,8 all end in 3 or 9, only two of those three start numbers will be used, and the 3 and 9 will also come from those. Therefore 4 and 6 must be in any selection, and must be followed by 1 and 7.
In summary, two of the numbers must start with 2,5,8 and end with 3,9 (6 possibilities), and two must start with 4,6 and end with 1,7 (2 possibilities). Overall there are therefore 6x2= 12 possibilities.

250.17 – Self-Assembly - Elliott Line

Authors of detective fiction:
(RAYMOND) CHANDLER
(AGATHA) CHRISTIE
(DASHIELL) HAMMETT
(JO) NESBO
(RUTH) RENDELL
(MICKEY) SPILLANE

250.18 – Sixth Powers - Elliott Line

All sixth powers are either a multiple of 13 or one away from a multiple of 13:

1 = 13x0 +1
64 = 13x5 -1
729 = 13x56 +1
4096 = 13x315 +1
5^6 = 13x1202 -1
6^6 = 13x3589 -1
7^6 = 13x9050 -1
8^6 = 13x20265 -1
etc.

To check that it works for ALL sixth powers, you only need to check the first 13, since:
(n+m)^a = (n)^a (mod m)

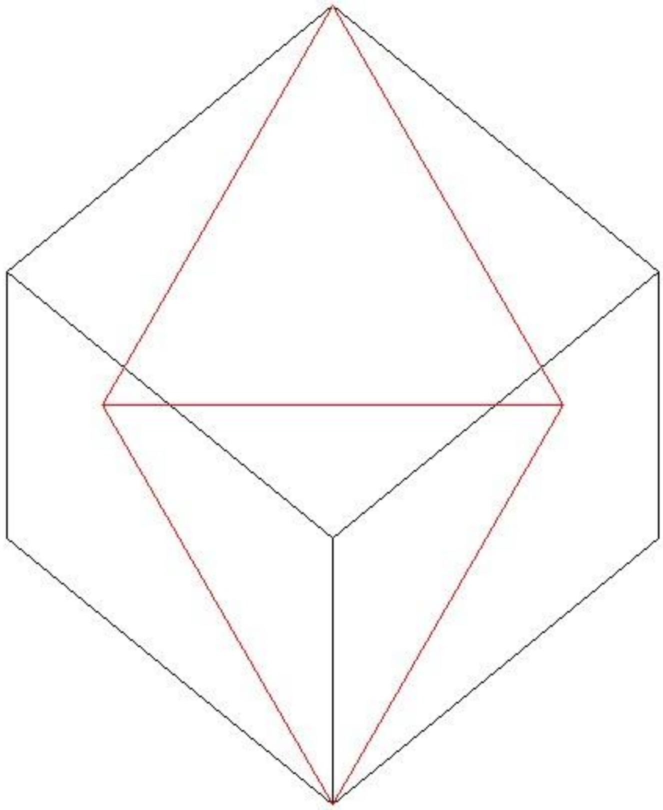
Proof: if you multiply out the left-hand side, one term will be (n)^a, plus several other terms which will all contain m, and therefore be a multiple of m, and not affect its value, modulo m.

In actual fact you only have to look at the first 6, since (m-n)^a = (-n)^a (mod m), for precisely the same reason.

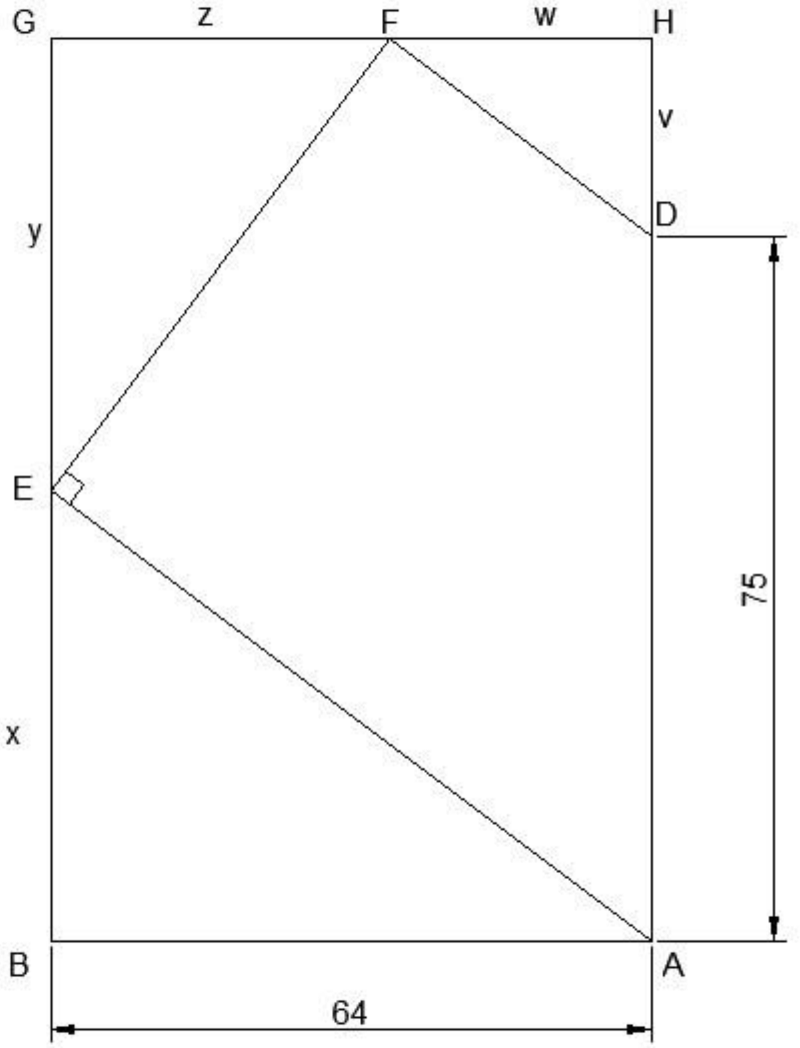
To be sure that there are no numbers higher than 13 that work, you only need to look at the first couple of sixth powers (and the numbers either side of them) and see what their factors are.
The only numbers that divide into (63, 64, OR 65) AND (728, 729 OR 730) are 1,2,3,4,5,7,8,9,13.
13 is the highest of this list, so as long as it proves to be a valid solution, it will be the highest such solution.

250.19 – Skeleton Cube - Elliott Line

When viewed such that the cubes where the spheres are centred are at the top and bottom of the figure, those points are square root of 3 apart. The unit lengths lines from the top and bottom vertices describe the extent of the overlap of the two spheres (they are actually cone shaped but from this angle appear to be equilateral triangles). The circle in the middle of the cube is viewed here edge-on, and so its diameter is 1.



250.20 – Triangle and Rectangle – Elliott Line



You can remove the line CD as it is not really helping, and instead introduce another line, parallel to CD, but passing through F.
On the face of it, there are too many degrees of freedom to be able to determine anything useful. HOWEVER, since we want DF to be a minimum, we can also add in the fact that angle EFD must be a right angle, and that lengths x and y must be equal.
Looking at the three similar triangles around the outside we can tell that:
$64/x = y/z = w/v$
As well as the facts that:
$x = y$; $z + w = 64$; $2x - v = 75$

We can either do a bit of algebra and find a cubic in terms of x, which factorises to one real and two complex solutions.
Or we can try a few values for x and see what works. It turns out that x=y=48, z=36, w=28, v=21, and therefore DF = 35.

250.21 – Unit Square - Elliott Line
When the two free endpoints are 0.414213… (sqrt(2) - 1) away from the lower left / upper right corner respectively, the shaded area will be at a maximum, which will be exactly 3.5 - sqrt(8), or 0.67157…, a little over two thirds.

250.22 - Uprooted - Elliott Line

Let √x=c

Let √y=d

The two given equations become:

$c^2d+cd^2=14$

$c^3+d^3=22$

If you cube (c+d), you will get an expression purely based on the two above equations:

$(c+d)^3 = c^3+d^3+3c^2d+3cd^2$

$(c+d)^3 = 22 + 14*3$

$(c+d)^3 = 64$

(c+d) = 4

If you add together the two equations:

$c^3+d^3+c^2d+cd^2=22+14$

$(c+d)(c^2+d^2) = 36$

But (c^2+d^2) is actually the (x+y) we are seeking, and we have already determined that (c+d)=4, so:

(x+y) = 36/4 = 9