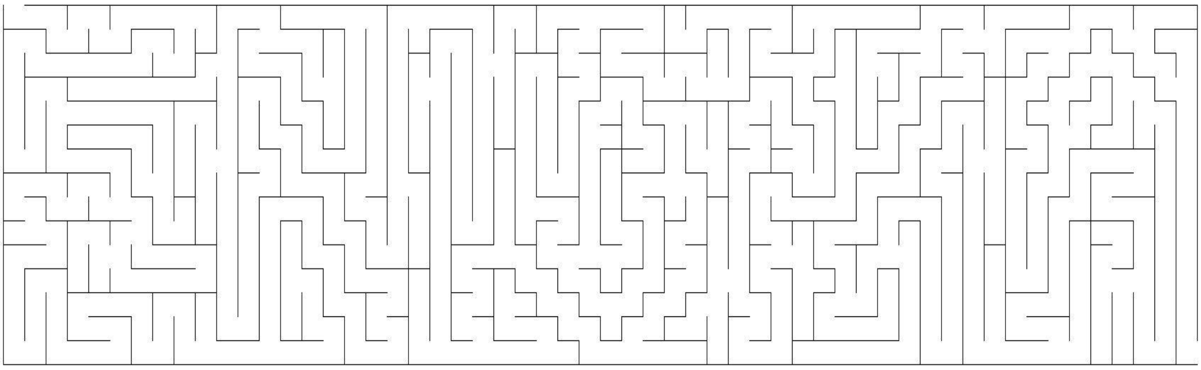
Enigma:
The Newsletter of PuzzleSIG



ENIGMA

251

'Mathematical problems, or puzzles, are important to real mathematics, just as fables, stories, and anecdotes are important to the young in understanding real life'.

Terry Tao

March 2022

Electronic Version of this Newsletter
Email enigma.mensa@yahoo.co.uk and I'll send you a copy

About Enigma
Enigma is the newsletter of Puzzle SIG.
The SIG for anyone interested in puzzles. The scope covers word puzzles, crosswords, logic puzzles, Japanese puzzles, mathematical brain teasers, lateral thinking problems, quizzes and picture quizzes, discussion of physical / mechanical puzzles, computer / internet based puzzles and puzzle games, and puzzle books and publications, and experimentation and innovation of new puzzle forms.

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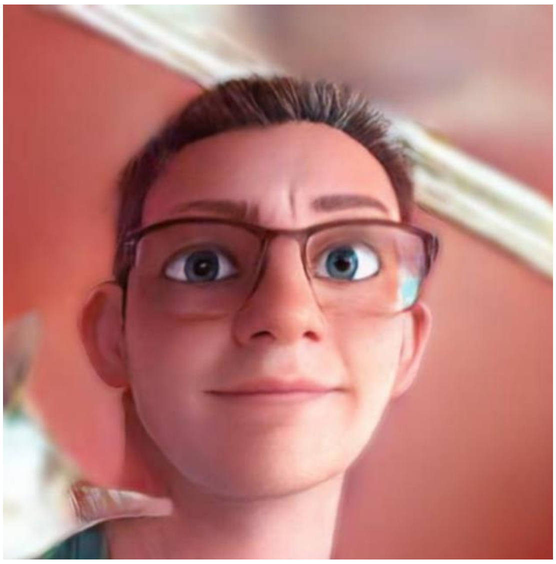
How to Join
You can join Puzzle SIG by visiting mensa.org.uk/sigs (member login required).

Welcome to Enigma 251

Hello and welcome to another issue of Enigma.

It's mostly maths and logic puzzles as usual, but with a couple of word puzzles at the end, one from Stephen Colbourn, SIGSec of Codes and Ciphers SIG, which I can recommend if you are interested in cryptography, ancient and modern.

Remember there is always the maze on the front cover too!



Please try all of the puzzles, and if you have any feedback, let me know and I'll pass it on to the puzzle's creator.
As ever, if you get stuck and need a hint, drop me a line and I'll be happy to help.
Please keep your puzzles, answers, comments, queries, suggestions, etc coming in.

Happy puzzling
Elliott.

250.01 - COMPETITION: Cycling Laps - Elliott Line

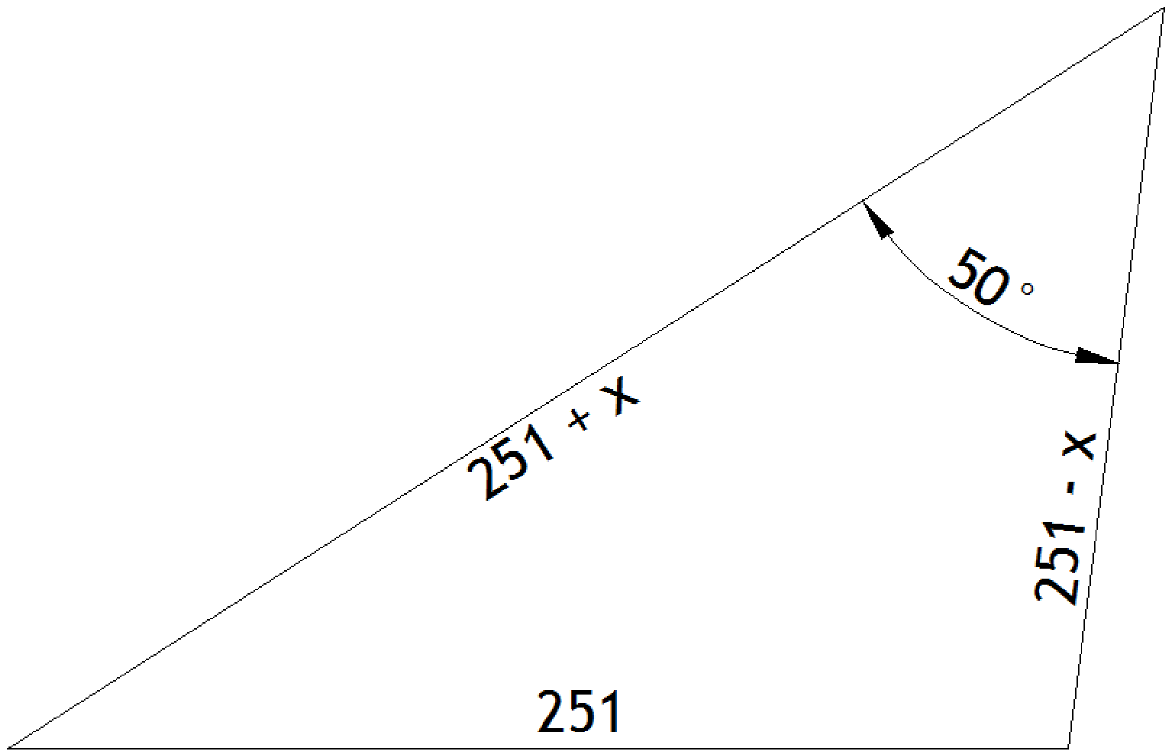
They will pass a marker every 1/9 of a kilometre, as follows:
139km, 28km, 167km, 56km, 195km, 84km, 223km and 112km

WELL DONE TO: **Paul Cook** **Karl Billington** **Johann Muller** **Michael Ryan** **Ivor Cornish** **Paul Clark** **Alan Davenport** **Matthew Francis** **Paul Bostock** **Jon Abbott** **Stuart Nelson** **Christa Ramonat**

251.01 - COMPETITION: More or Less - Elliott Line

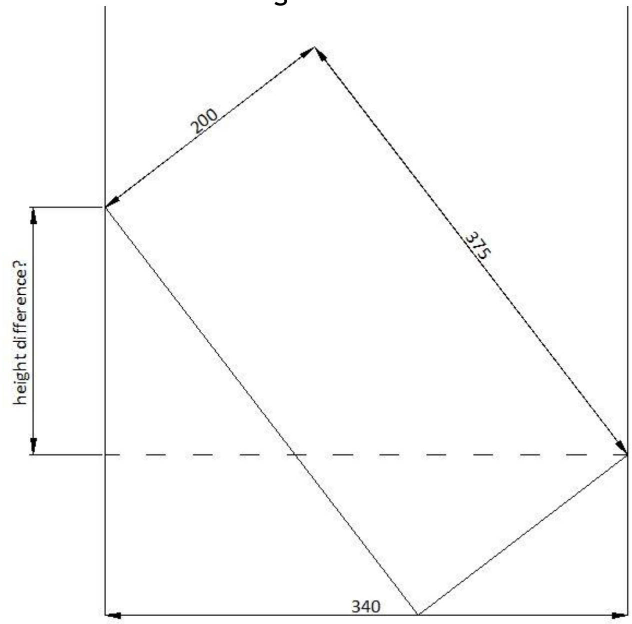
I have a triangle: its base is 251 long (251 being the number of this issue of Enigma). The other two sides are respectively longer and shorter than the base by the same whole number amount, which we'll call 'x'.
The angle opposite the 251 base is 50 degrees (give or take 1/5000th of a degree).

What is the value of 'x'?



251.02 – Block in the Alley – Elliott Line

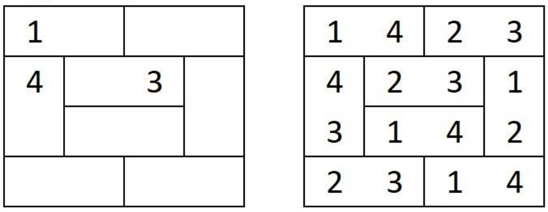
A rectangular block measuring 375cm x 200cm is positioned in a 340cm wide alleyway as shown. How much higher is the left corner than the right?

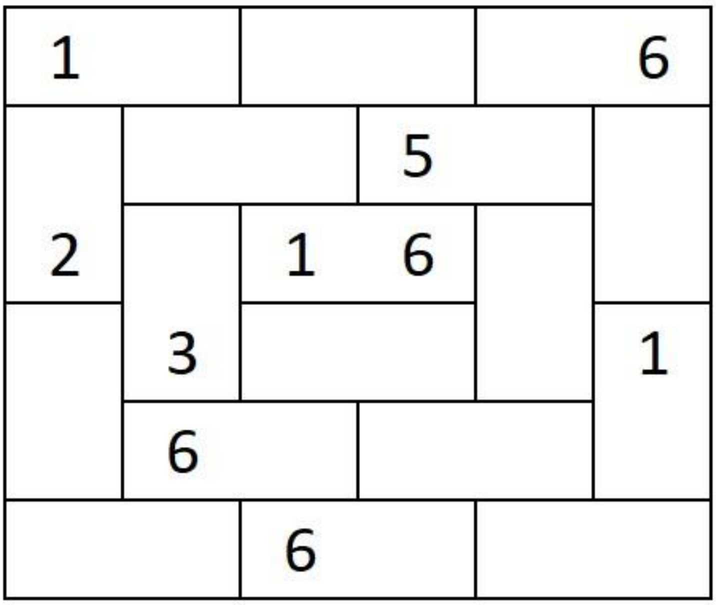


251.03 – Brickdoku - Elliott Line

Each row and each column should contain every number from 1 to 6 (1 to 4 in the example). Additionally, each ‘brick’ should contain one even number and one odd number.

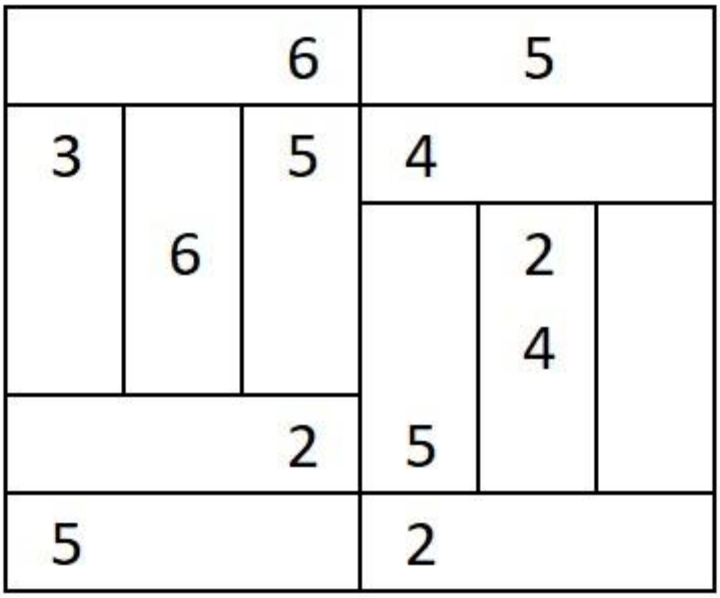
eg





251.04 – Brickdoku 2 - Elliott Line

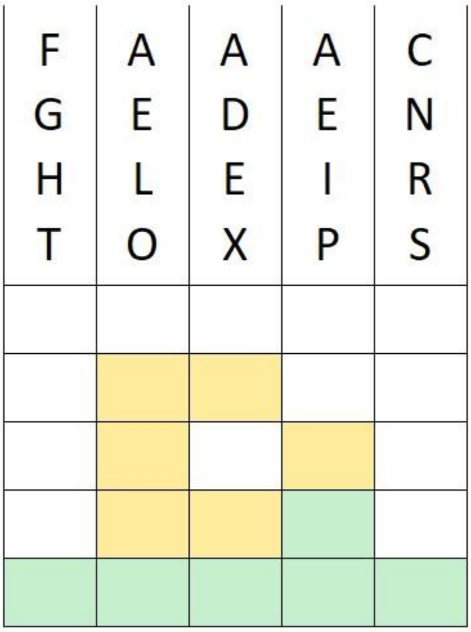
Each row and each column should contain every number from 1 to 6. Additionally, the numbers in each 'brick' should add up to a multiple of 3.



251.05 – Construct-a-Wordle - Elliott Line

You have probably seen this game going around the interwebz: The object is to guess the secret word. For each guess you make, you will be told if a letter is correct (green), or if it does occur in the secret word, just not where you've placed it (yellow), or if it's not in the secret word at all (white).
Not one to miss a bandwagon, I thought I'd make a variation on the theme:

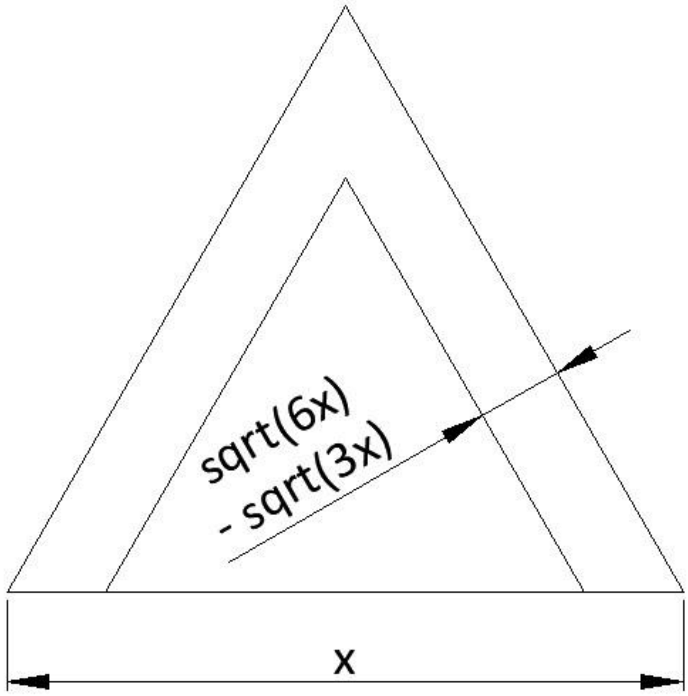
T	R	A	C	E
S	H	I	N	Y
M	O	U	L	D
P	I	A	N	O



In the Wordle to the left, the letters for the first four guesses are given, and are above the correct column. The puzzle in threefold: firstly reconstruct what the first four guessed words were; secondly, place the four words in the correct order in the grid; thirdly, use the green, yellow and white squares from the first four rows to work out what the actual secret word should be.

251.06 – Corridor - Elliott Line

There is a building whose plan view is an equilateral triangle, x feet along each side. The building is divided into an internal room and a corridor along two sides of the building, such that the area of the corridor is exactly the same as the area of the internal room. The width of the corridor is sqrt(6x) - sqrt(3x) feet. What is the value of x?



251.07 – Three Sisters - Elliott Line

There are three sisters, all of different ages, but all sharing a birthday.
At some point in the distant past, the older sister's age was a cube number and her sisters' ages were both prime numbers.
Now, some time later, the older sister's age is again a cube number, and the younger sisters' ages are again both prime numbers.
How old are they all?

251.08 – Counter Game - Elliott Line

I have 16 counters. On each turn I either remove 1, 2 or 3 counters until there are no counters left.
I could remove 2 counters eight times, or I could remove five lots of 3 counters then a single counter, or I could remove 3, 2, 2, 1, 3, 1, 1, 1, 1, 1, or any number of different ways.
Well in fact not 'any' number of different ways. Your task is to find out exactly how many ways there are. Note: the same combination but occurring in a different order counts as different ways, for instance, there is only one way using 2 eight times, but there are six ways involving five 3s and a 1.
I've chosen the number of counters to be 16 because the total number of ways is the square of a prime number. This fact doesn't particularly help you, except for checking whether or not you are correct.
How many different ways are there?

251.09 – Counter Game 2 - Elliott Line

In a similar vein to my previous puzzle Counter Game, there are a number of counters on the table, but this time you can either remove 2 or 7 of them on each turn. For instance if there were 11 counter there are three solutions: 2,2,7; 2,7,2; 7,2,2; (as before removing 7 then 2 is distinct from removing 2 then 7). Some numbers of counters have no solution, for instance if there are 5 counters you cannot remove them all.
If the number of counters is high enough, there are always more different ways to remove all the counters than to leave one counter behind, but this is not true for some smaller numbers, for instance if there were 12 counters there is only one way of removing them all (2,2,2,2,2,2), but as we have seen, three ways of removing 11 and leaving 1 behind.

What is the highest number of counters, for which there are fewer ways to remove them all than to leave one remaining?

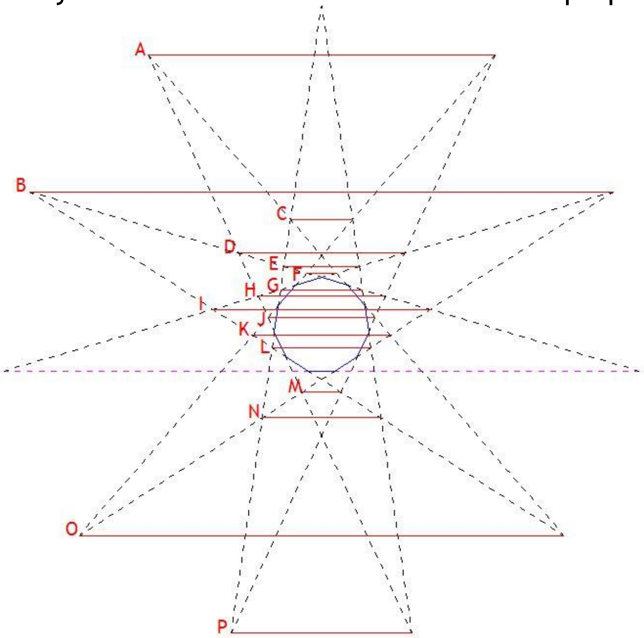
251.10 – Fiendish Sudoku - Elliott Line

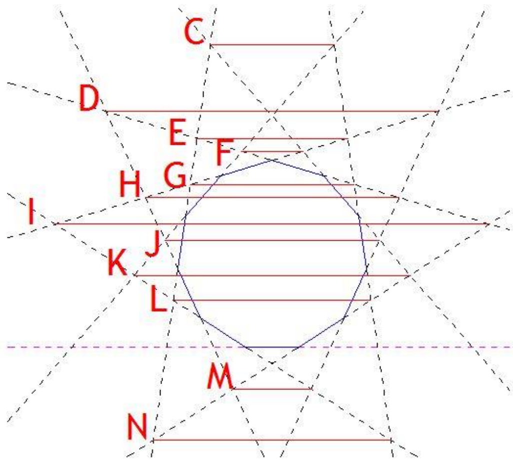
In the following sudoku, within each and every 3x3 'box', the rows add up to 9, 15 and 21 and the columns add up to 9, 15 and 21 (in some order).
For instance one box might be:
2 3 4
7 5 9
6 1 8
as the row totals are 9, 21 and 15, and the column totals are 15, 9 and 21.
Other than that, usual sudoku rules apply: the numbers 1 to 9 appear once each in every row, column and box.
Starting tip: list all the possible ways of totalling 9 then look at the central box.

			1					
	8							
					3			
			2			8		
		5						
								2

251.11 – Hendecagon Star – Elliott Line

I have taken a regular hendecagon (11-sided polygon) and added extensions to each of the sides so that they intersect as shown. Wherever it is possible to connect two of these intersection points with a horizontal line I have done so, and denoted them A to P. (Except for the magenta dashed line, which has several intersection points on it, but can be ignored for the purposes of this puzzle).
Some of these lines A to P have a very curious property, that if you form a circle whose diameter exactly coincides with said line, the circle will pass exactly through the midpoints of two of the sides of the original hendecagon.
Can you list all those lines which have this property?





Close-up view of centre of figure

251.12 – Human Logic - Elliott Line

Construct a list of 9 common five-letter words, such that with every new word in the list uses at least one letter that you haven't used in any of the preceding words. Just to make it a little more difficult, you may only use the letters in the words HUMAN LOGIC.

251.13 – Making the Infinite Finite - Elliott Line

What is the value of the following sum?

1/4 + 1/10 + 1/18+ 1/28 + … continuing forever,

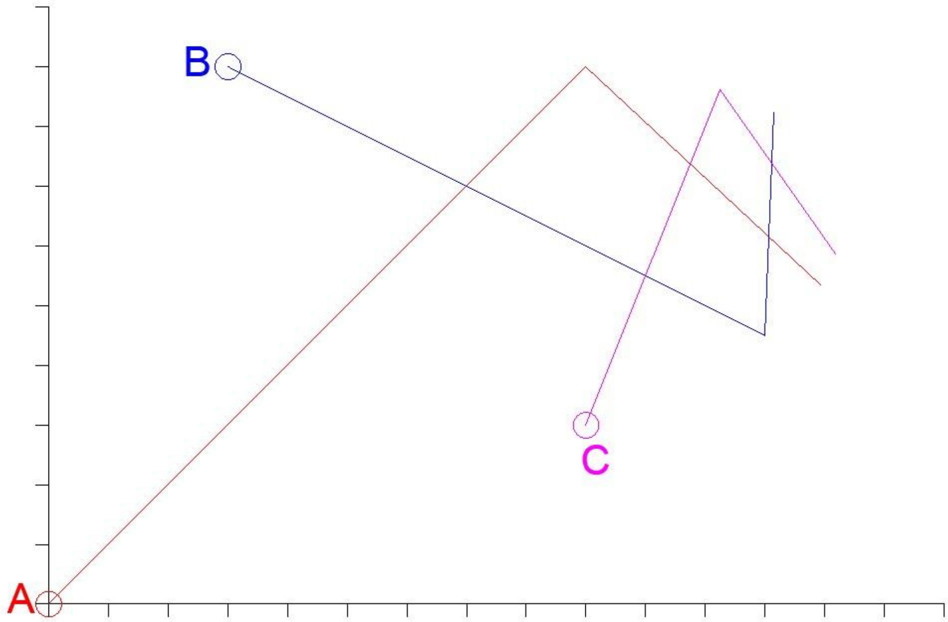
where each denominator is each successive natural number, n, multiplied by n+3.

251.14 – Infinite Coin Push - Elliott Line

We used to play a game back when I was at school. We would start with three coins placed widely apart on a table. The first player would give one of the coins a shove, so that it would pass between the other two. Then the next player would try to do the same, starting from the current position, but using one of the other coins. This would continue until one player would inevitably fail to get the coin between the other two.

I'm now imagining an idealised version, where the coins are single points, and on each turn the coin passes through the exact midpoint of the positions of the other two coins and continues a further 50%. So to begin with, the coins are at (0,0) (3,9) and (9,3) respectively. The midpoint of the initial positions of coins B and C is clearly at (6,6), so coin A will travel from (0,0) to (6,6) and then continue another 50% to come to rest at (9,9). Next it is the turn of coin B. Coin B must travel to the current midpoint of coins A and C and extend another 50% distance to find its new position. The coins take turns in the order A, B, C, A, B, C etc

The diagram below shows the first two moves of each coin.
After an infinite number of moves, where will each of the three coins be?



251.15 – Mastermind - Elliott Line

A code of three coloured pegs is selected from Red, Yellow, Green, Orange, Blue and Purple.

Guess 1: Red Yellow Green: one correct but in the wrong position

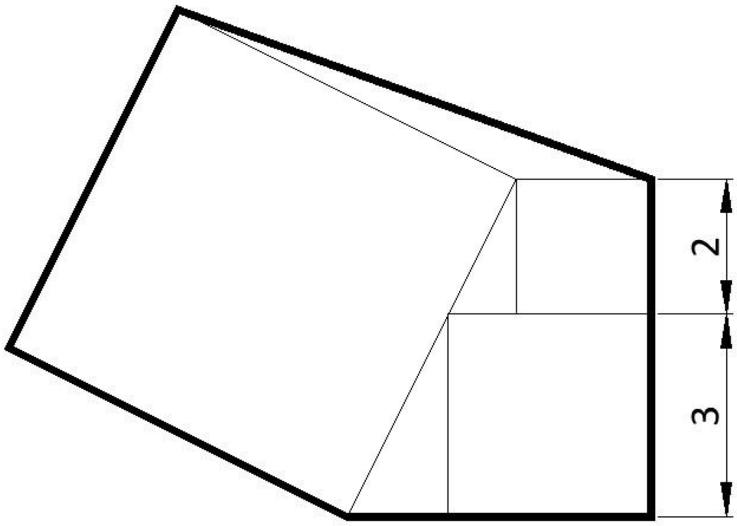
Guess 2: Orange Blue Purple: one correct but in the wrong position

Guess 3: Red Red Purple: one correct, and in the correct position

Solution: ?

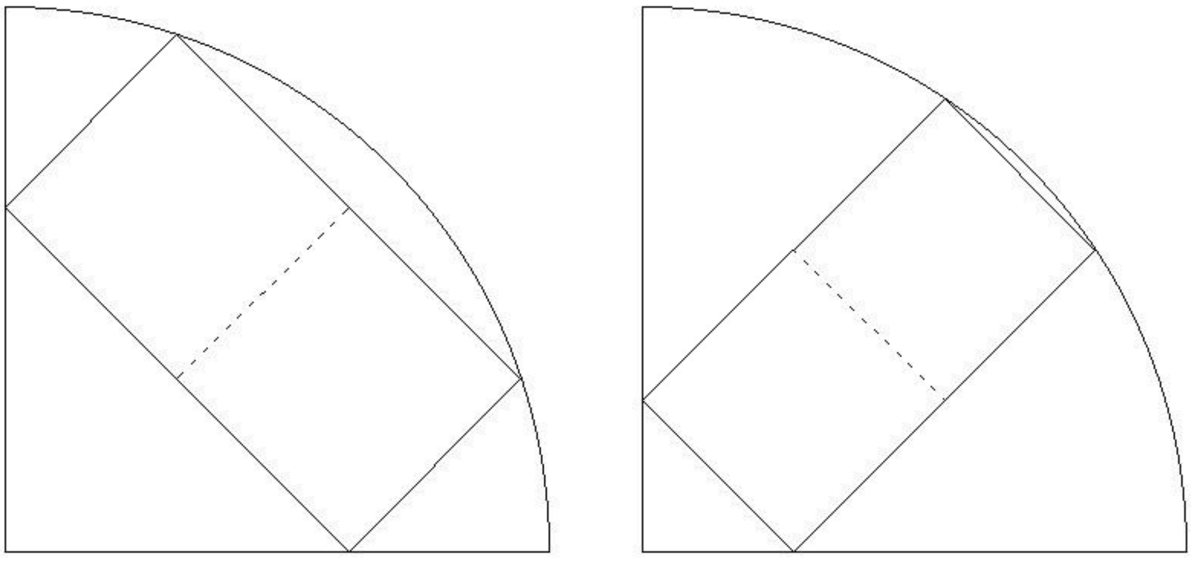
251.16 – Odd Shape - Elliott Line

Within this figure are three squares and three triangles. The edge length of two of the squares is 2 and 3 units respectively. What is the area of the overall shape?



251.17 – Rectangles in a Quadrant - Elliott Line

There are two ways to draw a rectangle whose length is twice as long as its width, within a quarter circle of radius 1, such that all four corners lie on the boundary of the quarter circle.
One of the rectangles has an area greater than the other, but by how much exactly?



251.18 – Secret Message - Elliott Line

Complete the final column to find a secret message:

G	A	S	M	A	H	
L	E	N	O	A	T	
V	E	R	U	T	N	

A	K	U	T	I	O	
E	U	R	A	R	N	
U	C	S	A	D	I	

M	E	R	M	C	R	
D	O	N	O	A	C	

251.19 – Word Jumble: Boring British Soup - Stephen Colbourn (Code and Ciphers ISIGSec)

Fit the solved anagrams into the grid and find a dreary British soup made from hooves and trotters. The first clue is given as an example.

1		B	A	U	B	L	E
2							
3							
4							
5							
6							
7							
8							
9							
10							
11							
12							

01. showy trinket or leabub
02. a plough makes a worfur
03. a rolled parchment or lorcsl
04. an alloy of tin, copper, and lead – treewp
05. an act causing general outrage – nasalcd
06. an orange or yellowish-brown colour – wytan
07. distinguishing qualities or characteristics – artist
08. following a winding course – renamed
09. a threadlike appendage of a climbing plant – derlint
10. a shepherd or church minister – sproat
11. colour and dye from Cr – ohmrec
12. marked with a cut or blemish – carders

~~~~~SOLUTIONS~~~~~SOLUTIONS~~~~~

251.02 – Block in the Alley – Elliott Line

You can use Pythagoras' theorem to find the length of the diagonal of the block to be 425cm. This diagonal is simultaneously the hypotenuse of another triangle whose base is 340cm and whose height is the number we are after. Using Pythagoras again we find that the height difference is 255cm.

251.03 – Brickdoku - Elliott Line

1	2	4	3	5	6
3	1	2	5	6	4
2	4	1	6	3	5
6	3	5	2	4	1
5	6	3	4	1	2
4	5	6	1	2	3

251.04 – Brickdoku 2 - Elliott Line

1	2	6	3	5	4
3	1	5	4	6	2
4	6	3	1	2	5
2	5	1	6	4	3
6	4	2	5	3	1
5	3	4	2	1	6
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251.05 – Construct-a-Wordle - Elliott Line

This is my solution. Well done if you found another solution which fits all of the facts.

T	O	X	I	C
H	E	A	P	S
F	A	D	E	R
G	L	E	A	N
E	Q	U	A	L

251.06 – Corridor - Elliott Line

x is equal to 32 feet, such that the corridor is a little over 4 feet wide.

251.07 – Three Sisters - Elliott Line

They are 64, 61 and 59.
An even number of years must have past between the two scenarios, since at least one of the sisters must have had an odd age in the earlier scenario (there is only one even prime). So therefore the cube numbers must be of the same parity. They can’t both be odd, since 1 is not bigger than any prime numbers, and 125 exceeds a human lifetime, so 27 is the only viable odd cube. So the older sister must have been 8 in the early scenario and 64 now.
There are four prime number below 8: 2,3,5 and 7. Adding 56 years to these gives 58, 59, 61 and 63. Only 59 and 61 are prime, so these are the younger sisters’ ages now.

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251.08 – Counter Game - Elliott Line

10609 different ways. This is 103 squared.
Trying to go ahead and systematically list all of the ways is going to be a massive task, but luckily there is a quicker way.
The first go can either be 1, 2 or 3. If it was 1 then there would be 15 counters left; if it was 2 there would be 14 counters left; if it was 3, 13 left. All quite obvious so far. So if we only knew how many ways there were of combining to make 13, 14, and 15, we could just add them together to find the total number of ways for 16. We can do this safely knowing there is no overlap but that they cover all cases, as they all start with different numbers. All very nice, but the trouble is we don't know how many ways there are of making 13, 14 or 15. Luckily our rationale will work in general, and for any number of board spaces, we only have to add up the previous three to find the answer. This holds until the number of counters is less than four, but luckily those numbers are more manageable.
1:1 way
2:2 ways (1+1 or 2)
3:4 ways (1+1+1, 1+2, 2+1, 3)
Building on this, knowing that each subsequent number of ways is simply the sum of the previous three, we can soon find the answer:
4:7
5:13
6:24
7:44
8:81
9:149
10:274
11:504
12:927
13:1705
14:3136
15:5768
16:10609
There is even a clever way to avoid working out the first few, that works in general. For each negative number there are 0 ways, and for zero there is 1 way, then every subsequent number of ways is the sum of the previous n. So for instance if n=6, ie. if you had a six-sided dice and wanted to know how many different ways of reach exactly 10 with multiple throws:
0:1
1:1
2:2
3:4
4:8
5:16
6:32
7:63
8:125
9:248
10:492

As an extra bonus question, does this sequence, the one based on a six-sided dice, ever yield a prime number of ways (apart for the 2 ways of getting 2)?

251.09 – Counter Game 2 - Elliott Line

This can be solved using exactly the same method as the first Counter Game puzzle. For all negative numbers, the number of ways is 0, and for 0 it is 1, and each subsequent number n, the number of ways is the sum of the number of ways of n-2 and n-7.

0:1
1:0
2:1
3:0
4:1
5:0
6:1
7:1
8:1
9:2
10:1
11:3
12:1
13:4
14:2
15:5
16:4
17:6
18:7
19:7
20:11
21:9
22:16
23:13
24:22
25:20
26:29
27:31
28:38
29:47
30:51
31:69
32:71

For 25 counters the number of ways is less than for 24. By coincidence n=25 is also the last time that the number of ways is less than n. To know for sure the sequence doesn't reverse again after this point, it is only necessary to find 7 in a row that are increasing, hence me continuing the sequence to n=32.

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251.10 – Fiendish Sudoku - Elliott Line

5	7	9	1	2	6	4	3	8
3	8	4	5	9	7	2	1	6
1	6	2	3	4	8	9	5	7
2	1	6	4	8	3	7	9	5
9	5	7	2	6	1	8	4	3
4	3	8	9	7	5	6	2	1
7	9	5	6	1	2	3	8	4
6	2	1	8	3	4	5	7	9
8	4	3	7	5	9	1	6	2

Staring with the central box, notice that 9 must be formed in different ways in both the vertical and horizontal directions. There are only three ways this is possible using the numbers 1 to 9 without repetition: 1,2,6; 1,3,5; 2,3,4. Since we know 2 and 3 are not in the same column or row, we know 1 must be in the same column and row as 2 or 3. Column 4 already has a 1, so the 1 can be placed in column 6, row 5. To complete the sums to 9, the 5 and 6 can be placed. By attempting to form the sums to 15 and 21 you will find there is only one way to place the remaining numbers in the central box. The next leap of insight is that for every horizontal sub-row, the 9 must be 1+2+6, the 15 3+4+8 and the 21 5+7+9. And for each vertical sub-column, 9 = 1+3+5, 15 = 2+4+9, 21 = 6+7+8. Any attempt to deviate from this will quickly lead to repetition of digits. It’s still not straightforward, but it is at least possible to logically deduce the remaining cells, broadly starting with the top middle box and moving anti-clockwise.

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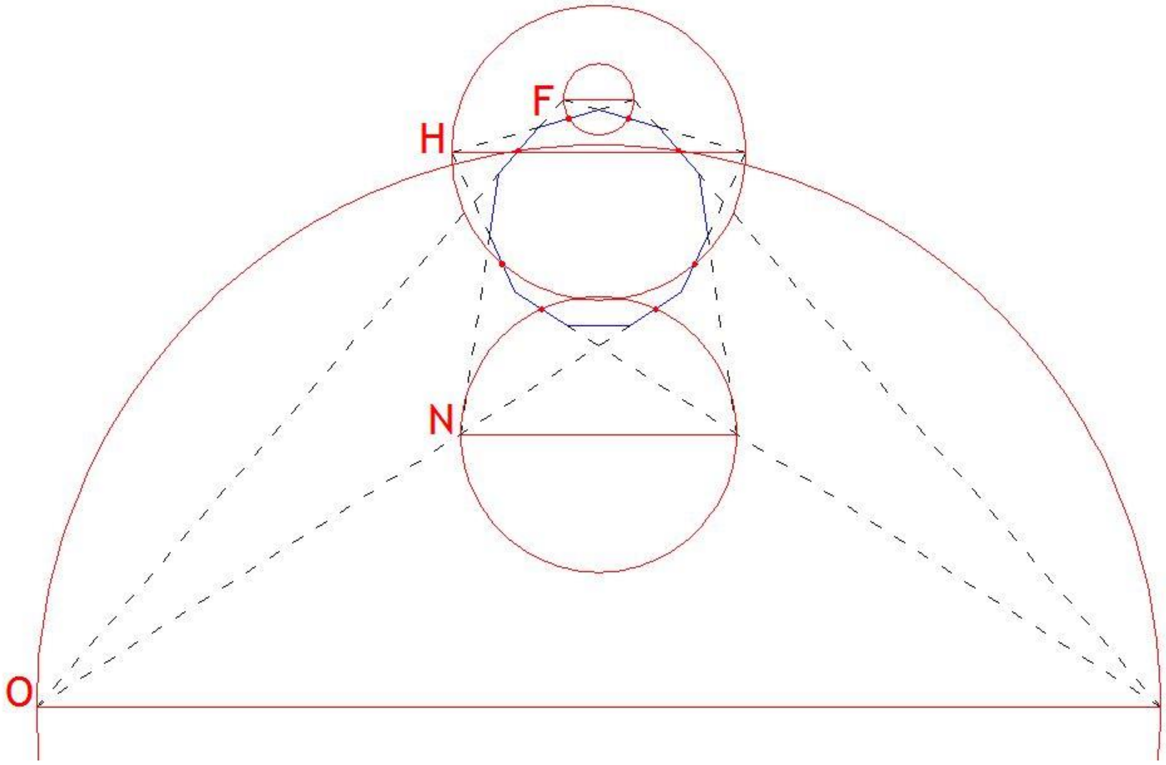
251.11 – Hendecagon Star – Elliott Line

Lines F, H, N and O are all diameters of circles which pass through midpoints of sides of the original hendecagon.

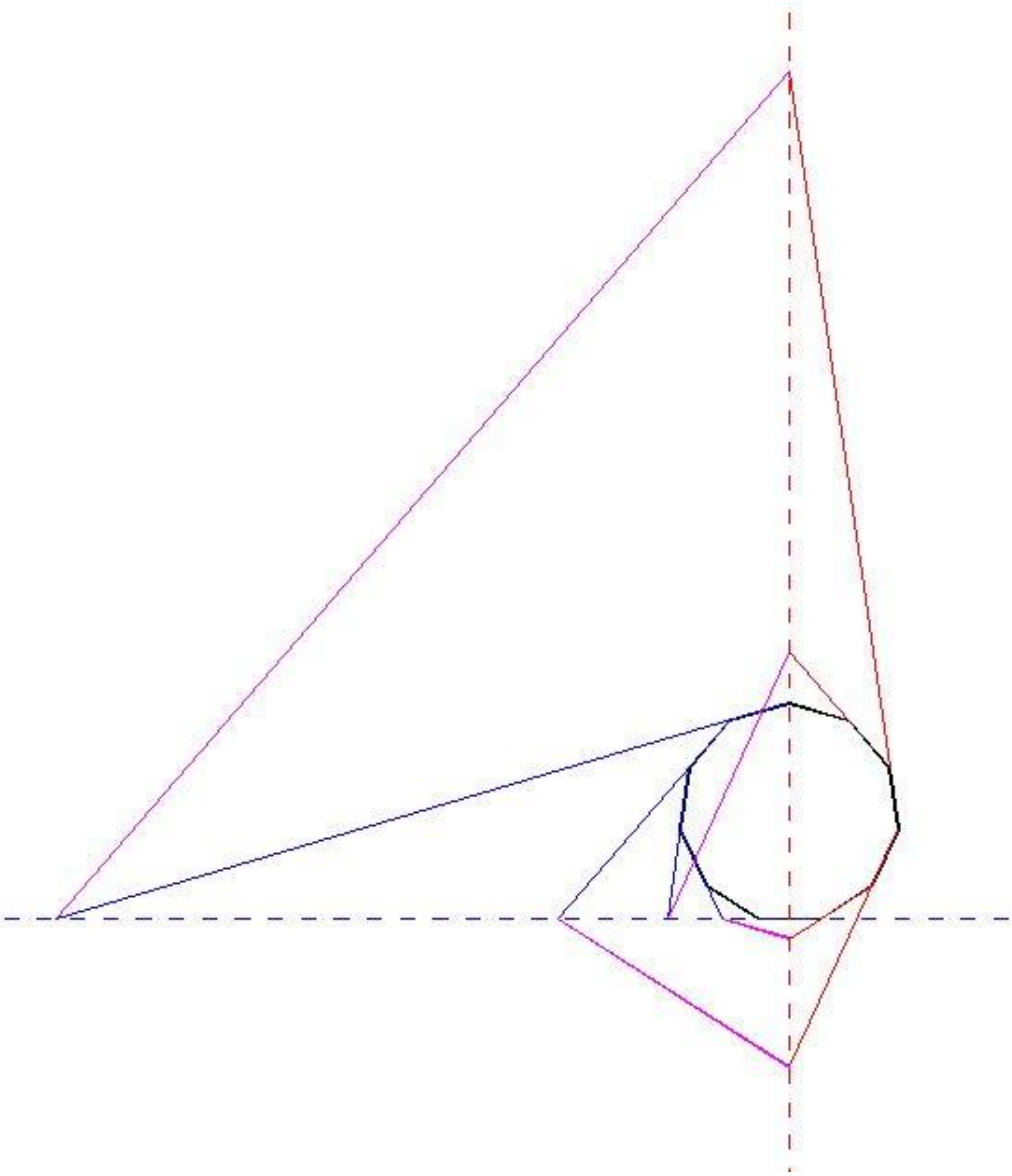


The key insight is that an angle within a semicircle will be a right angle, so we start off with two construction lines at right angles, one collinear with one of the sides of the 11-sided polygon, another passing through the vertex opposite that side. By definition they will intersect at the midpoint of the side of the polygon.

On the left hand side draw extensions from all possible polygon edges that can meet the blue (horizontal) line, and on the right hand side draw extensions from all possible polygon edges that can meet the red (vertical) line (some will go upwards, others downwards). The red intersections are intersection points of extended polygon sides, as the red lines can be mirrored by the red axis. The blue intersections are already intersections of extended polygon sides as the blue axis itself is an extended polygon side.

The intersection points with the main axes can be paired up according to how far they are from the centre of the polygon. By definition any circle with one of these magenta pairing lines as its diameter will pass through the point where the axes meet (as well as a second polygon midpoint which will vary from circle to circle).

None of these pairing lines are horizontal as in the puzzle, but it’s easy to see how for each pairing line, the figure can be rotated such that the line is horizontal. If you do so you will find that the four lines correspond to the lines F, H, N and O in the puzzle.



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251.12 – Human Logic - Elliott Line

This puzzle will necessarily have several possible answers; here is one I came up with. To reach 9 words with only 10 different letters, each word in the list should only introduce one new letter, with the exception of the first word, although even that needs to be limited to just two different letters.

MAMMA MA
MAGMA G
LLAMA L
HALAL H
LAUGH U
GHOUL O
AMIGO I
HUMAN N
LOGIC C

~~~~~SOLUTIONS~~~~~SOLUTIONS~~~~~

251.13 – Making the Infinite Finite - Elliott Line

Each term 1/(n(n+3)) can be rewritten as two separate terms:

A/n + B/(n+3)

To find the values of A and B we force them back into a single fraction and make sure it's equal to the original fraction:

(A(n+3) + Bn)/ (n(n+3)) = 1/(n(n+3))

(A+B)n + 3A = 1

The n term must disappear as there isn't one on the right-hand side, so A+B=0.
3A=1, so A=1/3, and so B=-1/3

So each term 1/(n(n+3)) is equivalent to 1/3n - 1/(3(n+3)). For example the first term 1/4 is equal to (1/3 - 1/12), the second term 1/10 is equal to (1/6 - 1/15), etc.

This might seem to be unnecessarily complicating the infinite sum, however you might notice that the negative part of each term will appear again as the positive part, three terms later:

(1/3 - 1/12) + (1/6 - 1/15) + (1/9 - 1/18) + (1/12 - 1/21) + (1/15 - 1/24) + (1/18 - 1/27) …

Virtually everything will cancel out. Only the positive parts of the first three terms will remain, and can easily be added together to find the final answer:

1/3 + 1/6 + 1/9 = 11/18, or 0.6111…

If you were to add up successive terms on a spreadsheet you would find that it does indeed approach this sum, but it does so very very slowly. It only reaches 0.6 after about 90 terms, 0.61 after about 900 terms, 0.611 after about 9000 terms, 0.6111 after about 90000 terms etc.

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251.14 – Infinite Coin Push - Elliott Line
All three coins will be at (13,7).
The key to solving this is to compute the x and y coordinates separately. We are given the first three x coordinates, 0, 3, 9 and each subsequent x coordinate will be 3/4 of the previous two less 1/2 of the one before that. Eventually this settles on an x coordinate of 13. Doing the same for the y coordinates will eventually settle on 7.

In general, given starting positions of A, B, C, the coins will converge on: (-2A+B+4C)/3.

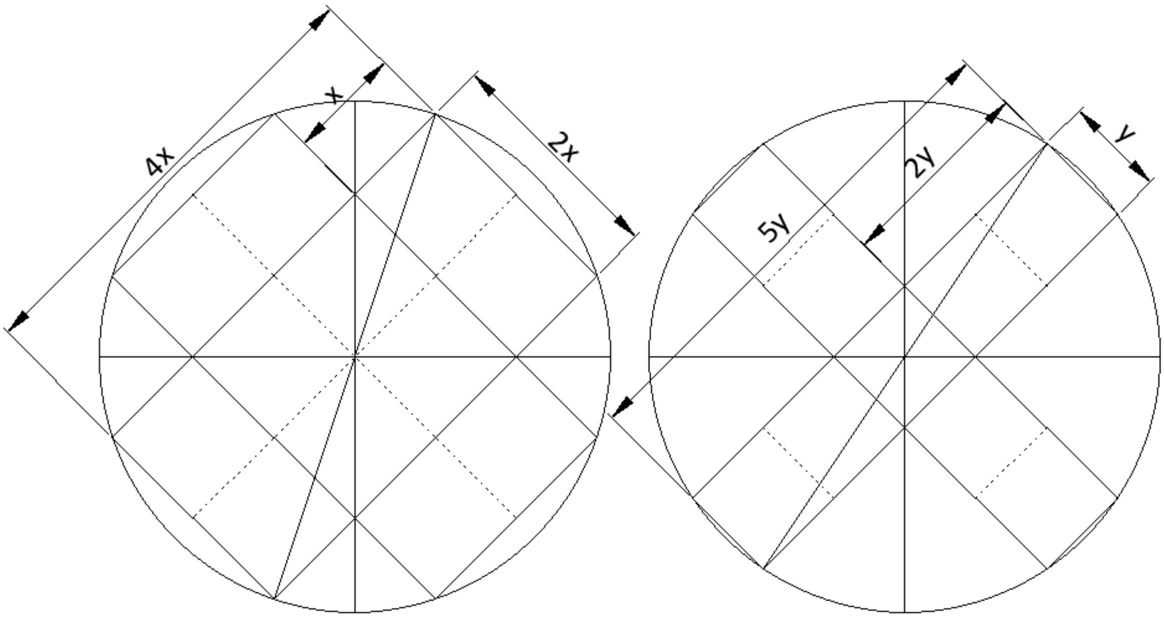
251.15 – Mastermind - Elliott Line
From guess 3, one is in the correct place. It cannot be the initial Red as that would have shown as in the correct place in guess 1. Similarly it cannot be the final purple, as guess 2 would have shown it. So it must be the red in position 2.
Since the first two guesses cover all of the colours, but only show two correct pegs, there must be a colour repeated. It cannot be the red, or else that would have shown in guess 3. It must be one of the colours from guess 2, but since they must occupy positions 1 and 3 in the answer, they cannot be Orange or Purple, as they would have shown as in the correct place in guess 2.
The final answer is therefore Blue Red Blue

251.16 – Odd Shape - Elliott Line
The squares have area 4, 9 and 31.25 respectively, and the triangles have areas of 2.25, 1 and 2.5. The total area is 50 square units.

~~~~~SOLUTIONS~~~~~SOLUTIONS~~~~~

251.17 – Rectangles in a Quadrant - Elliott Line

The simplest way to calculate the areas of the rectangles is probably to complete the circles, with three additional copies of each quadrant, then draw a diameter between two opposite points as shown:



In the first case the area of the required rectangle is 2x^2, and in the second it is 2y^2.

In the first diagram, there is a right triangle with legs 2x and 4x, and hypotenuse 2.

(2x)^2 + (4x)^2 = 2^2

4x^2 + 16x^2 = 4

20x^2 = 4

2x^2 = 4/10 = 2/5

Similarly in the second diagram:

y^2 + (5y)^2 = 2^2

26y^2 = 4

2y^2 = 4/13

So the areas are 2/5 and 4/13 respectively, and their difference is therefore 6/65. Or if you chose to interpret the question the other way, the first rectangle is 30% larger than the second.
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~~~~~SOLUTIONS~~~~~SOLUTIONS~~~~~

251.18 – Secret Message - Elliott Line

Within each block, countries can be read diagonally downwards, jumping back to the top after the bottom row of the block. The message reads: YOU ARE OK.

G	A	S	M	A	H	**Y**
L	E	N	O	A	T	**O**
V	E	R	U	T	N	**U**

A	K	U	T	I	O	**A**
E	U	R	A	R	N	**R**
U	C	S	A	D	I	**E**

M	E	R	M	C	R	**O**
D	O	N	O	A	C	**K**

251.19 – Word Jumble: Boring British Soup - Stephen Colbourn

'Brown Windsor'

1		B	A	U	**B**	L	E	
2		F	U	R	**R**	O	W	
3		S	C	R	**O**	L	L	
4			P	E	**W**	T	E	R
5		S	C	A	**N**	D	A	L
6			T	A	**W**	N	Y	
7		T	R	A	**I**	T	S	
8		M	E	A	**N**	D	E	R
9		T	E	N	**D**	R	I	L
10			P	A	**S**	T	O	R
11		C	H	R	**O**	M	E	
12		S	C	A	**R**	R	E	D