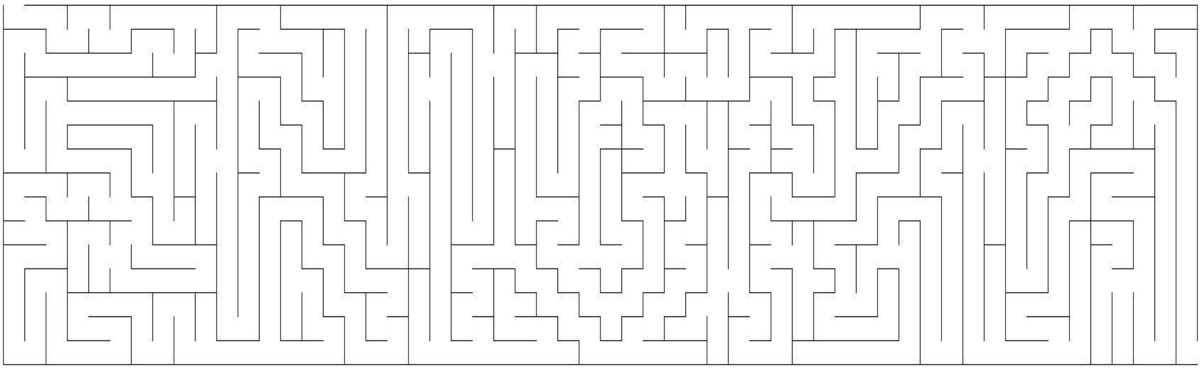
Enigma:
The Newsletter of PuzzleSIG



ENIGMA

253

'Mathematical problems, or puzzles, are important to real mathematics, just as fables, stories, and anecdotes are important to the young in understanding real life'.

Terry Tao

December 2022

Electronic Version of this Newsletter
Email enigma.mensa@yahoo.co.uk and I'll send you a copy

About Enigma
Enigma is the newsletter of Puzzle SIG.
The SIG for anyone interested in puzzles. The scope covers word puzzles, crosswords, logic puzzles, Japanese puzzles, mathematical brain teasers, lateral thinking problems, quizzes and picture quizzes, discussion of physical / mechanical puzzles, computer / internet based puzzles and puzzle games, and puzzle books and publications, and experimentation and innovation of new puzzle forms.

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How to Join
You can join Puzzle SIG by visiting mensa.org.uk/sigs (member login required).

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Welcome to Enigma 253

Hello and welcome to another issue of Enigma.

Hope this reaches you well, and hopefully you can enjoy these challenges over the festive season.

Please don't feel put off by the fact that all of the puzzles in this issue are written by me, or that most of them are maths-based. I personally prefer setting maths puzzles, and so that is what I tend to set, but if you are reading this as a reader then you also qualify as a potential contributor, and I would love to include any of your puzzles you want to sent to me, especially if they are not maths-based, as that would serve to balance out the content of the newsletter. By the same token, if you're happy just to try to solve the puzzles set by me and the other contributors, that's fine too!

There were only two correct answers to the competition puzzle from the last issue, superb job Chris and Agnijo! Let's see if more people can solve the next one on page 4.

Speaking of Agnijo Banerjee, he hasn't asked me to say this, and I hope he doesn't mind, but I highly recommend the series of books he has written with David Darling: Weird Maths, Weirder Maths and Weirdest Maths, available from all good stockists.

Please try all of the puzzles, and if you have any feedback, let me know and I'll pass it on to the puzzle's creator.
As ever, if you get stuck and need a hint, drop me a line and I'll be happy to help.
Please keep your puzzles, answers, comments, queries, suggestions, etc coming in.

Happy puzzling
Elliott.

252.01 - COMPETITION: Starshaped - Elliott Line

CE is 84mm, exactly 1/3 of BH.

WELL DONE TO:
Chris Craig
Agnijo Banerjee

253.01 - COMPETITION: Reversals - Elliott Line

23 divided by 32 equals 0.71875 precisely.

However, 23 isn't the only two-digit number which, when divided by the reversal of its digits is equal to 0.71875. 46/64 and 69/96 would also both work. This is clear as in each case the numerator and the denominator are multiplied by the same factor.

When looking at three-digit numbers, 253 (being the number of this issue of Enigma) turns out to be the ONLY number which, when divided by its reversal is equivalent to 23/32. 253/352 = 0.71875. Once again the numerator and denominator are merely 11 times the original 23 and 32.

There are nine examples within the four-digit numbers, but can you find them all?

This is a competition, but not for prizes, only bragging rights. Every correct answer I receive will get an honourable mention in the next issue of Enigma. Send your answer to me at enigma.mensa@yahoo.co.uk .

253.02 – Coin Paradox – Elliott Line

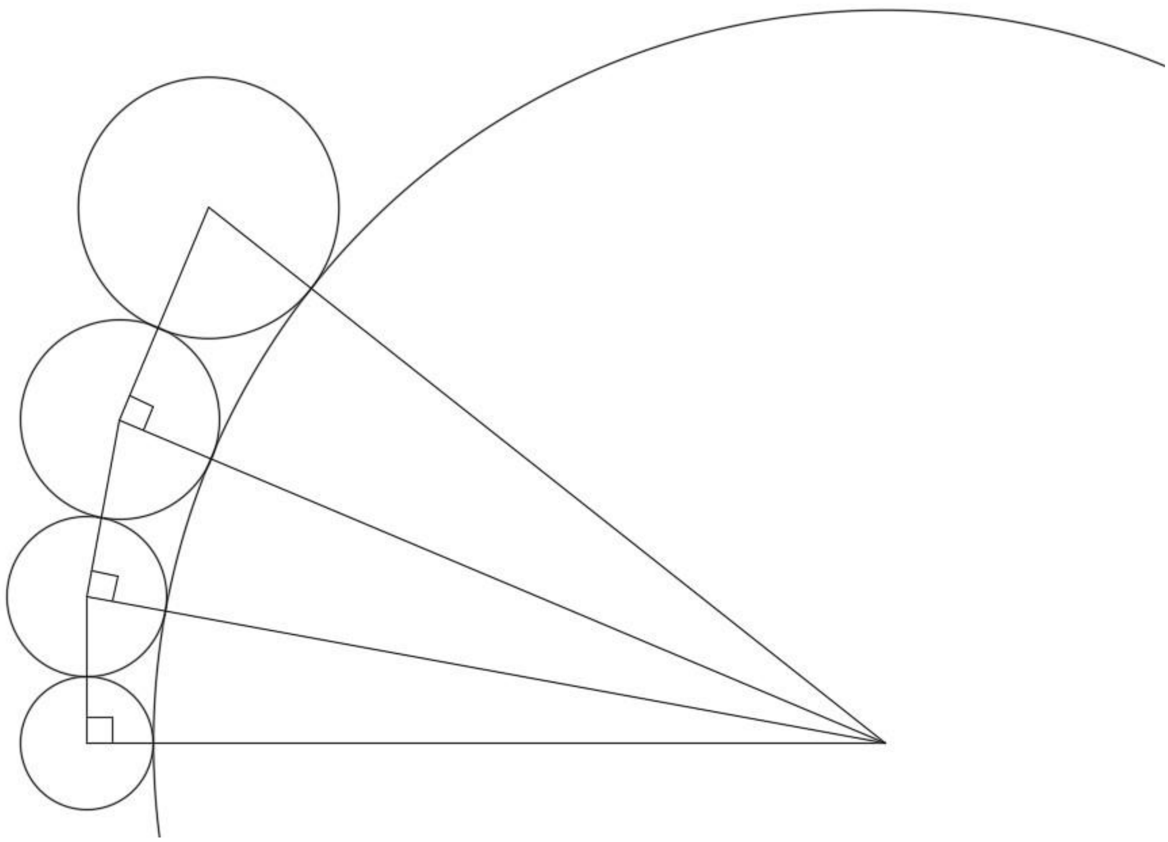
I'm thinking about a sequence of coin flips resulting in either heads or tails, with equal probability. Which of the following statements are true?

In a game where I win if we flip HTH on consecutive throws and my friend wins if we flip HTT on consecutive throws:

1) If we just toss the coin three times in succession, we each have an equal probability of winning (although most of the time neither of us would win).
2) If we keep flipping until either HTH or HTT comes up, we will win the game with equal probability.
3) If we keep flipping until either HTH or HTT comes up, the average number of flips it will take is on average the same, whether I win or my friend does.
4) If we each have our own coin, and I keep flipping until I see HTH on consecutive flips, and my friend keeps flipping her coin until she sees HTT on consecutive flips, we will both take the same number of flips on average.

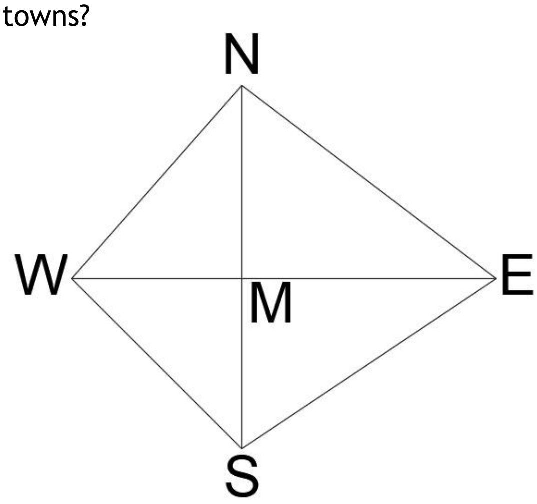
253.03 – Five Circles – Elliott Line

I have five circles arranged tangent to one another as below, and the triangles formed by connecting some of their centres result in three right-angled triangles.
Can you find an arrangement where the radius of each of the five circles is a whole number?
For bonus points, what is the minimum arrangement where each of the radii is a whole number (minimum sum of the five radii)?



253.04 – Five Towns – Elliott Line

The towns of Norton, Sutton, Weston, Easton and Middleton are arranged such that the Norton-Sutton road and the Weston-Easton road cross at right angles in Middleton.
The distance from any town to any other town is unique.
Each distance is an exact whole number of miles.
What is the smallest possible total area of the diamond shape formed by the outer four towns?



253.05 – Mystery Numbers – Elliott Line

a, b, c, d are positive integers.

The sum of products of triples is equal to the sum of products of pairs:

abc+abd+acd+bcd = ab+ac+ad+bc+bd+cd = X

What is the value of X?

253.06 – Odd Prime Cycle – Elliott Line

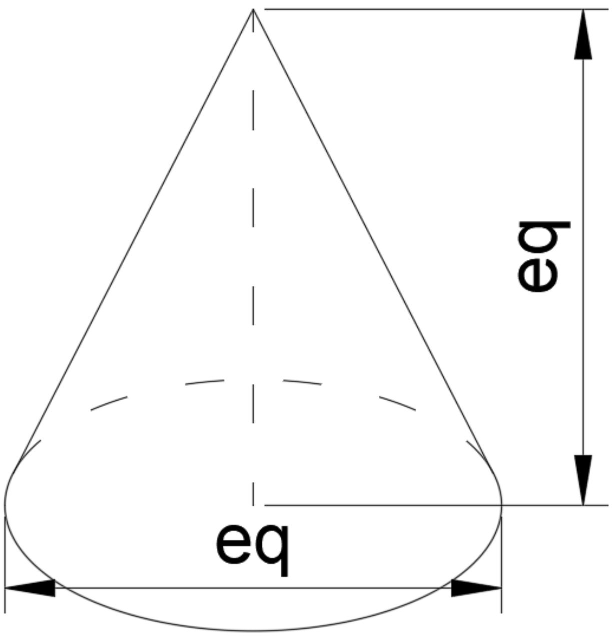
Start with an odd prime number
Add 1, then choose an odd prime factor of this new number,
Add 2 to that number, then choose an odd prime factor of this new number,
Add 3 to that number, then choose an odd prime factor of the new number,
Add 4 to that number, then choose an odd prime factor of the new number,
etc, until the odd prime factor you get is the odd prime you started with.

For instance, if you started with 5:
5+1 is divisible by 3
3+2 =5

Other than 5, what could the starting odd prime number have been?

253.07 – Optimal Cone – Elliott Line

I have a right cone, whose base diameter is equal to its height. Its surface area exceeds its volume, and the difference between surface area and volume is at a maximum. What is the surface area?



253.08 – Prime Balance – Elliott Line

I have some identical coins positioned around the edge of a flat disc, such that:

The coins are in several distinct piles, and the piles are equally spaced around the edge of the disc.
Each pile contains a prime number of coins.
No two piles are the same size.
The centre of gravity of the coins is in the exact centre of the disc.

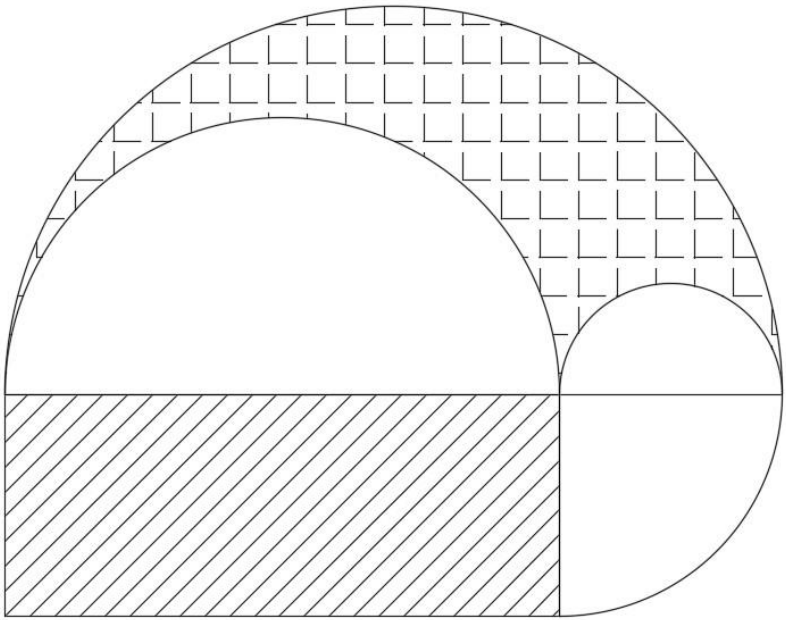
What is the minimum number of coins I could have?

253.09 – So Many Digits – Elliott Line

I start with the number 74.
I append just enough 3s to the start so that the new number 3…374 is divisible by 74.
Finally I append just enough 1s to the start so that the new number 1…13…374 is divisible by the previous number 3…374.
How many digits do I now have altogether?

253.10 – Shaded Areas – Elliott Line

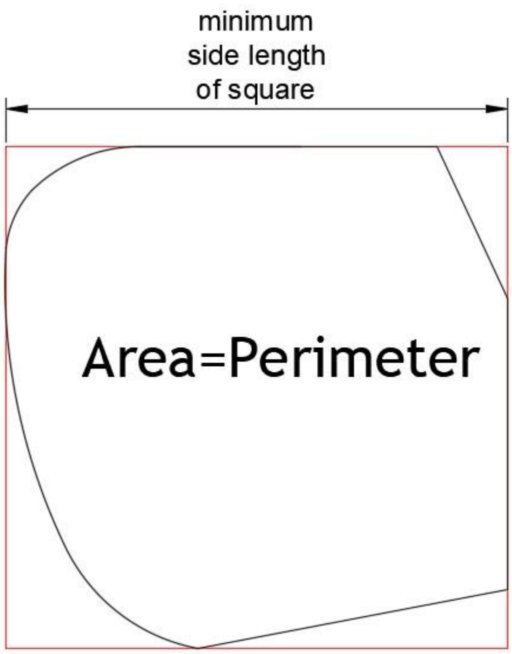
The figure below consists of three semicircles, a quarter circle and a rectangle.
If the area of the shaded rectangle is equal to 4, what is the area of the other shaded region?



253.11 – Smallest Square – Elliott Line

What is edge length of the smallest square that can contain a shape with equal (non-zero) perimeter and area?

Examples of such shapes are a circle of radius 2, which has both area and perimeter equal to 4*pi, or a 6,8,10 triangle, which has both area and perimeter equal to 24. My shape might be made up of straight lines, or curves, or both.



253.12 – Sorted! – Elliott Line

I am going to show you a sorting algorithm. In this example there are 9 letters, three each of A, B and C. They begin in a collated order: ABCABCABC, but they must end in a sorted order: AAABBBCCC in as few steps as possible. A 'step' involves selecting some portion of the string and reversing the order of the letters within it.
For our example this is possible in only three steps:
A[BCABCA]BC > A[ACBACB]BC
AA[CBA]CBBC > AA[ABC]CBBC
AAAB[CCBB]C > AAAB[BBCC]C

Now to the actual puzzle. This time you have 12 letters, which start collated and must finish sorted. I will let you decide how many different letters there are in the 12: either 2 each of 6 different letters, 3 each of 4 different letters, 4 each of 3 different letters, or 6 each of 2 different letters. Which of the following collated strings will take the fewest steps to sort?

ABCDEFABCDEF

ABCDABCDABCD

ABCABCABCABC

ABABABABABAB

253.13 – Strings – Elliott Line

I have a number that is a string of 1s, followed by a string of 2s, followed by a string of 3s. For example 111233. There must be at least one of each digit.
Similarly I have a second number that is a string of 3s, followed by a string of 2s then a string of 1s, with again at least one of each of the different digits. For example 332211111.
Adding these two numbers together I get the answer 44443444.
How many different possibilities are there for my two numbers?

253.14 – Two Point Oh Eight – Elliott Line

52/25 = 2.08 precisely
572/275 = 2.08 too

52 and 572 are the only two- and three-digit numbers which, when divided by the reversal of their digits, is exactly equal to 2.08.

How many 25-digit numbers are there, that when divided by their reversal become equal to 2.08?

This might look like an exercise in coding, but it isn't. In fact I devised the puzzle purely on paper, and it could be solved as such too.

253.15 – Ten Pin Bowling – Elliott Line

This game uses all 100 scrabble tiles, including the two blanks (which can represent any letter of your choosing).
The scoring system is the same as that of real ten-pin bowling: you get points for each word, equal to how many letters in the word. In addition, if you get a spare (use all letters in one frame using two words), you get bonus points equal to the next word you score.
If you get a strike (a ten letter word), you get bonus points equal to the next TWO words you score.
If you only get one word in a frame, and it's not a strike, then for the purposes of bonus points, you get a zero length word too.
In real tenpin bowling, if you get a strike or a spare on the tenth frame, you get an eleventh frame to determine your bonus points, and if you were lucky enough to get a strike on the tenth AND eleventh frames, you would get a twelfth frame.
In this game, there are no eleventh or twelfth frame, so to determine any bonus points you are entitled to after the tenth frame, look back at the words you scored in the first couple of frames.
The 100 scrabble letters have been assigned randomly into their frames as below (which means I can play along with everybody else). Generally a score of 100 or more using relatively common words is a pretty good benchmark, although a higher score is probably possible by poring over lists of more obscure words.

1
U E N O
A W R
T E
E

2
R F I I
L H Q
J R
I

3
O A I G
I S O
A P
Y

4
R A K V
Y ? E
I R
E

5
I T N O
C T X
A D
E

6
W R E D
T N B
O S
N

7
B E D ?
D S C
E E
O

8
U G S F
V N O
L L
T

9
I L A N
A E A
G T
M

10
U H Z M
U I P
E A
O

253.16 – Vaguely Defined Pentagon – Elliott Line

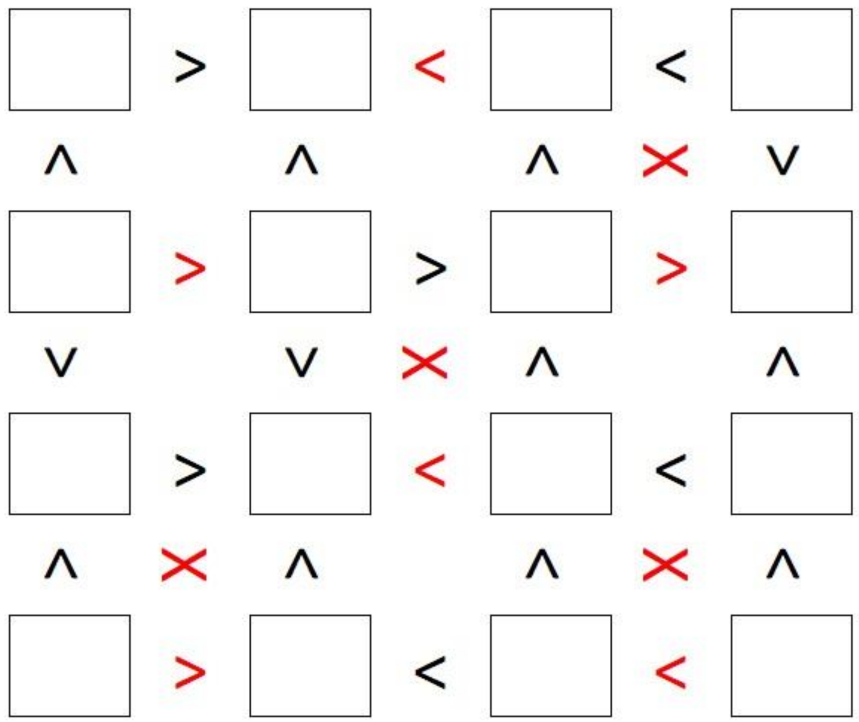
A pentagon has side lengths that are five consecutive whole numbers, arranged in numerical order around the pentagon.
Three of the internal angles of the pentagon are right angles.
What is the area of the pentagon?

253.17 – Weird Inequalities – Elliott Line

In what circumstance is 10 is higher than 7, but 1 is higher than 2, and 4 is neither higher nor lower than 8?

253.18 – Zipline Futoshiki – Elliott Line

You must place the number 1 to 16 in the grid such that:
An even number N must be placed in the same row or column as N-1.
An odd number M must be placed in the same diagonal as M-1.
(So if you were to trace the paths from 1 to 2, on to 3, etc all the way to 16, you would be alternating between rook moves and bishop moves).
All of the inequalities are correctly observed.
If the inequality is red, the numbers concerned are consecutive numbers.
A red X means that one or other of the pairs of numbers diagonally adjacent to the X are consecutive numbers.



SOLUTIONS:

253.02 – Coin Paradox – Elliott Line

Statements 1, 2 and 3 are true.

Statement 1: HTH and HTT will each happen 1/8 of the time.

Statement 2: at some point HT will come up on consecutive throws. The next throw will be H or T with equal probability and so HTH and HTT will have an equal chance of coming up.

Statement 3: the average number of throws to see HT is 4, and as we have seen, the game will finish on the next throw regardless of what it is, so the average number of flips ending in either HTH or HTT is 5.

Statement 4: this seems like it should be true, but it isn't. On average it will take me 10 throws to see HTH but my friend only 8 throws to see HTT. This is because if my friend reaches HT, she will either finish on the next throw, or have an H to start off a new attempt at HTT. If I'm on HT I will either finish on the next throw or be back at square one.

Interestingly, if we make the target sequences differ earlier, for instance HHH vs HTT, then even statements 2 and 3 will be false. HTT would win 3 times out of 5, and the average number of flips when HHH wins would be 5 2/5, whereas the average number of flips when HTT wins would be 5 11/15.

253.03 – Five Circles – Elliott Line

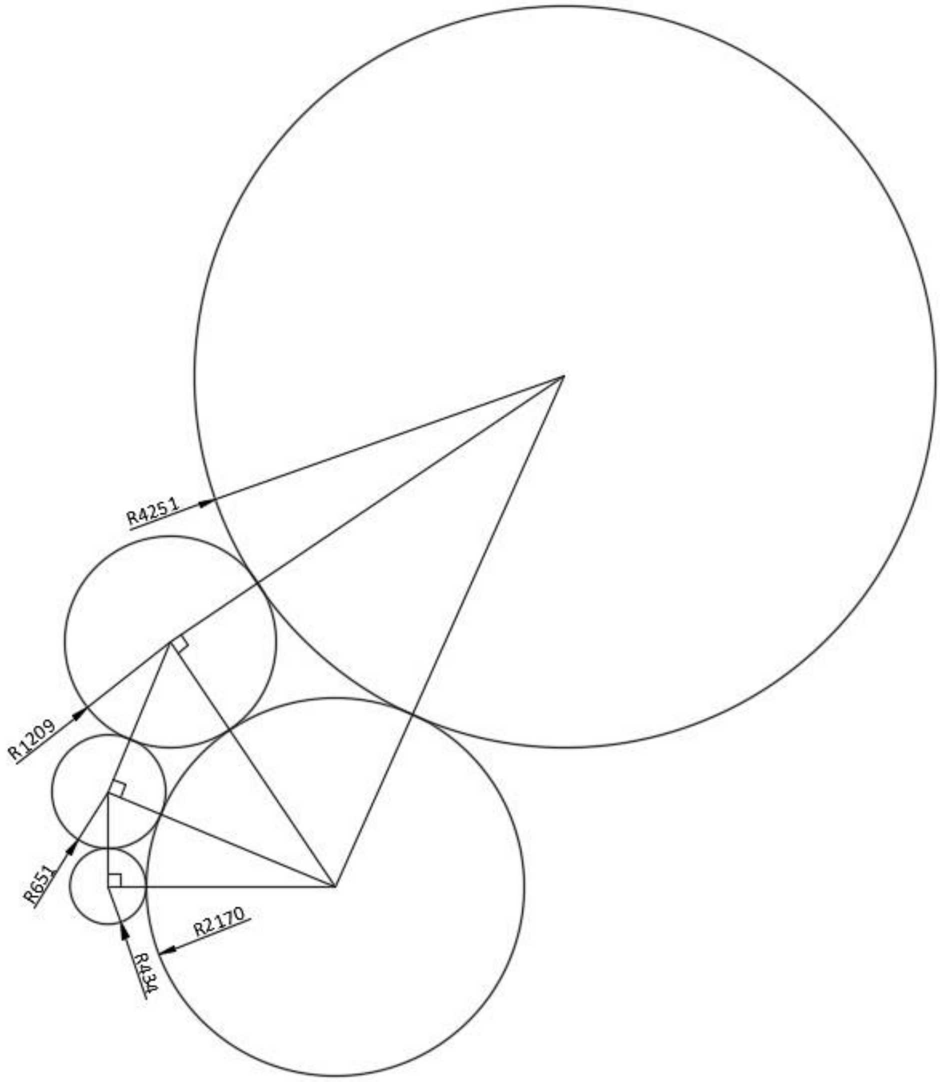
If you start with the smallest triangle and ensure it is a Pythagorean triple, then the radii of all of the circles is bound to be rational, and so can always be scaled up by an appropriate factor to obtain whole numbers. Beginning with a larger triangle and working backwards will not necessarily ensure this.

The best solution is found by starting with a 5-12-13 Pythagorean triple. The radii of the tangent circles for this are found by the semi-perimeter less each of the triangle edges, so 10-3-2.

Using Pythagoras on the second triangle, (10+3),(3+x),(x+10), we find that x needs to be 39/7. We scale up all the numbers we have by a factor of 7 to maintain whole numbers.

On the third and final triangle we now have (70+39),(39+x),(x+70), we find that x is 4251/31. Scaling everything up by a factor of 31, the five radii are:

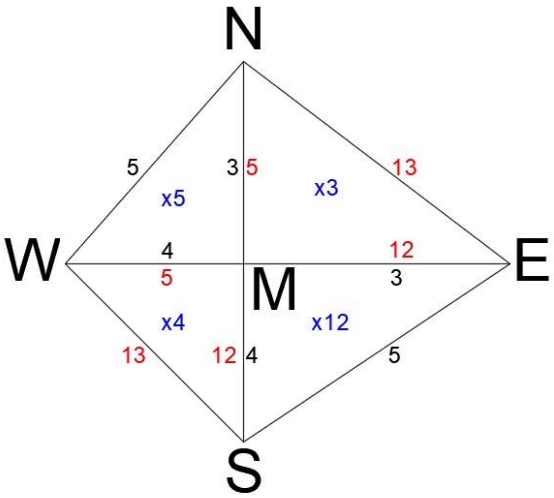
2170, 434, 651, 1209 and 4251, for a total of 8715.

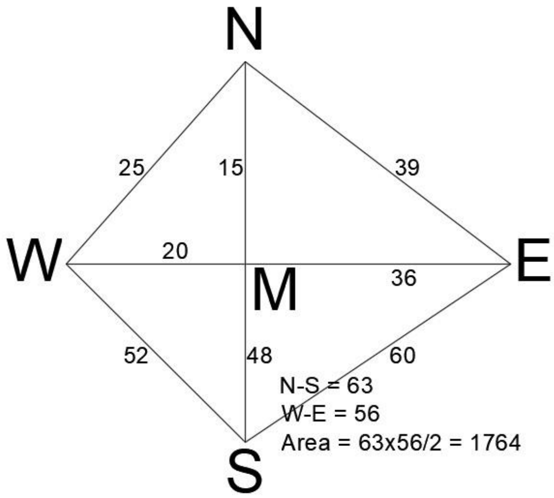


We might have used a different triangle to begin the construction, but we would have found that the procedure fails when the ratio between the radii of the zero-th circle and the first circle (2170 and 434 in our figure) is 4 or less. So for instance the 3-4-5 triangle would not have worked.

253.04 – Five Towns – Elliott Line

The most economical approach is for two opposite triangles to be scaled up versions of a 3-4-5 triangle, and the other two to be scaled up versions of the next smallest Pythagorean triple: 5-12-13. The area of the diamond is then half of the product of the two roads through Middleton:





253.05 – Mystery Numbers – Elliott Line

Let's first assume that all a,b,c,d are at least 2.
If they were all exactly 2, the triple sum would be 4x2x2x2=32, and the pair sum would be 6x2x2=24, so the triple sum is greater. If we increase any of the numbers, say we change a to 2+e, the triple sum would be increase to 32 + 12e , but the pair sum would only increase to 24 + 6e. In short, the pair sum can never catch up with the triple sum. So our assumption that all numbers were at least 2 is false. Without loss of generality we can let a=1. The equation then becomes:

$$bc + bd + cd + bcd = b + c + d + bc + bd + cd$$

And so $bcd = b + c + d$

Looking for a trio of positive integers whose sum is the same as their product, the only candidate is 1,2,3.
So if our a,b,c,d are (in some order) 1,1,2,3, the triple sum and the pair sum are both equal to 17.

253.06 – Odd Prime Cycle – Elliott Line
The first two possible starting numbers that eventually returns to themselves are 5 and 13:

As we saw in the example:
5+1 is divisible by 3
3+2 is divisible by 5

13+1 is divisible by 7
7+2 is divisible by 3
3+3 is divisible by 3
3+4 is divisible by 7
7+5 is divisible by 3
3+6 is divisible by 3
3+7 is divisible by 5
5+8 is divisible by 13

The third smallest is 14107.

Any other starting number less than 14107 will at some point reach a power of 2, which has no odd prime factors, without first getting back to the starting number.
For example, the longest sequence (for at some points you have a choice of odd primes) beginning with 101 is: 101 – 17 – 19 – 11 - 5 – 5 – 11 – 3 – 11 – 5 – 3 – 7 – 19 – (32).

253.07 – Optimal Cone – Elliott Line

The formulas for the surface area and volume of an equal right cone are:

Area = pi*r^2*(sqrt(5)+1)

Volume = 2/3*pi*r^3

We want the difference to be a maximum:

A-V = pi*r^2*(sqrt(5)+1)-2/3*pi*r^3

Let's differentiate:

pi*2r*(sqrt(5)+1)-2/3*pi*3r^2

2*pi*r*(sqrt(5)+1-r)

The (Area-Volume) will be at a maximum when the above expression is equal to 0, or more specifically, when the term in brackets is equal to 0, which is obviously when r = sqrt(5)+1.

To answer the original question, the surface area will be pi*(sqrt(5)+1)^3 ~ 106.464.

253.08 – Prime Balance – Elliott Line

The first thing to do is to establish the minimum number of piles. Clearly if there was 1 pile it would not be balanced around the centre of the disc. Two piles could be balanced but only if they were equal, which is not allowed. Three piles too can only be balanced if equal. Four can only be balanced if opposite pairs are equal.

Five is a little trickier. As we saw with the star balance puzzle it is in fact possible to have five different weights balanced around the centre. It turns out that it isn't possible for them all to be rational, so clearly can't all be primes.

So six piles seems our best bet. If you remember back to the pentadecagon balancing puzzle, we could split the weights into subsets of pentagons and triangles. With our six piles we can do something similar, but with triangles and opposite pairs. If we call our piles a to f, we need to find w,x,y,z such that each of the following is prime:

a=x
b=y+w
c=z
d=x+w
e=y
f=z+w

x, y and z each appear in the weights of opposite pairs, and w appears in an equilateral triangle of positions, so the system will balance whatever we choose for w,x,y,z.

Since x,y and z are all primes, and adding w to each gets new primes, w must be even. Trying 2, we need a trio of primes x,y,z that when each is increased by 2 we get three other primes, with no overlap.

Without any loss of generality we can say that $x<y<z$. If x=3, y cannot be 5, as x+w is already 5. It can't be 7 either as 7+2 isn't prime. We try y=11. z cannot be 13 but could be 17.

So with these values of w,x,y,z we have values of a to f of (3,13,17,5,11,19). These total 68 coins. Try as we might with other values for w,x,y,z, we would always need more than 68. And we can also eliminate the possibility of more than six piles: 7 will not give us a rational solution, for the same reasons that 5 won't. And any more than that, the total of the first n prime numbers already exceeds 68.

So the minimum solution is 68 coins, in piles of 3, 13, 17, 5, 11, 19.

253.09 – So Many Digits – Elliott Line

Glossary: a number which only contains 1s, such as 11 or 11111 is called a repunit. All prime numbers with the exception of 2 and 5 divide exactly into a repunit of p-1 digits, and frequently divide exactly into smaller repunits too (some divisor of p-1).

74 is 37 x 2, but we don't need to worry about the factor of 2, as 3…374 and 1…13…374 will naturally contain exactly one factor of 2 too.

3…374 can be rewritten as 2(1…1 x 150 + 37).

Since 150 and 37 do not share any factors, we are effectively looking for the smallest repunit that 37 divides into. By inspection 37 x 3 = 111, and so therefore our intermediate number will have three 3s: 33374.

For the second part we need to find the smallest repunit that 16687 (half of 33374) divides exactly into. The first thing we need to do is to decompose 16687 into its prime factors. 16687 = 11 x 37 x 41.

11 itself IS a repunit, specifically the second repunit, so the number of 1s we need must be even. From the fact that 37 divides into 111, the number of 1s must also be a multiple of 3.

We can use modular arithmetic to find the first repunit that 41 divides into:

1(mod41), *10+1=11
11=11(mod41), *10+1=111
111=29(mod41), *10+1=291
291=4(mod41), *10+1=41
41=0(mod41)

Therefore 41 divides into the fifth repunit 11111 (in fact it's 271 x 41), and so the number of 1s in the final answer will also be a multiple of 5.

Since these 2, 3 and 5 are relatively prime, their lowest common multiple is merely their product: 30. So our number will have 30 1s, followed by 33374, so 35 digits altogether:

11,111,111,111,111,111,111,111,111,111,133,374

253.10 – Shaded Areas – Elliott Line

If we let the dimensions of the rectangle be 2x and 2/x, then whatever the value of x, its area will be 4.

This means that the radii of the three semicircles will be:

x, 1/x and (x+1/x)

The area of a semicircle is (pi*r^2)/2, and we want the area of the larger semicircle, less the area of the smaller two. For simplicity we can take out the pi/2:

Area = pi/2 * ((x+1/x)^2 – x^2 – 1/x^2)
Area = pi/2 * (x^2 + 2(x/x) + 1/x^2 – x^2 – 1/x^2)
x gets cancelled out
Area = pi/2 * (2)
Area = pi
And that's the answer!

253.11 – Smallest Square – Elliott Line

Intuitively it makes sense that the basic shape of inner region to be a square with rounded corners. I'll leave it to the reader to try to prove this.

We need to know what the radius of the rounded corners needs to be.

We can do this by considering a circle, letting the radius vary, and finding when the value of (perimeter – area) is at a maximum.

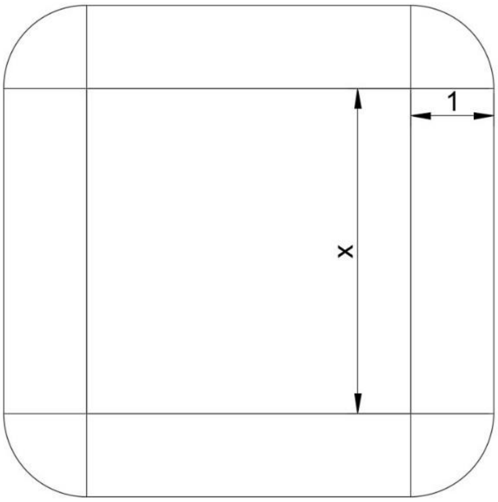
Perimeter = 2*pi*r

Area = pi*r*r

Perimeter – Area = r*(2-r)*pi

As pi is a constant, we just need the value of r where r*(2-r) is at a maximum. Clearly this is achieved when r=(2-r), so when r=1.

So now we need to find a square with radius=1 rounded corners, whose area is equal to its perimeter. The best way is to split the shape into regions:



The four quarter circles combine to form an entire circle, and then we have an inner square and four rectangles.

Area = pi + 4x + x^2

Perimeter = 2*pi + 4x

Making these equal tells us that x^2 = pi, so x = sqrt(pi).

The side length of the bounding square is therefore sqrt(pi)+2 = 3.772…

253.12 – Sorted! – Elliott Line

The string with three each of four letters (ABCDABCDABCD) can be solved in four steps; the other three options each take five steps.

A[BCDABCDA]BCD > A[ADCBADCB]BCD

AA[DCBA]DCBBCD > AA[ABCD]DCBBCD

AAAB[CDDCBB]CD > AAAB[BBCDDC]CD

AAABBBC[DDCC]D > AAABBBC[CCDD]D

253.13 – Strings – Elliott Line

There are two overall cases: the 3 digit in the sum could be made from a 1 in the first number and a 2 in the second number, or vice versa.

In the first case, that means the first four digits of the first number must also be 1s, and so the first four digits of the second number must all be 3s. We have no choice. However for the '444' at the end of the sum, we have some choice. The first of those 4s must be 2+2 (as otherwise the first number wouldn't have any 2s). The last digit must be made of 3+1. However the middle of those 4s could be either, giving us two possibilities.

If instead the 3 in the sum is made of 2 in the first number plus 1 in the second number, now the second half of the sum is completely determined and we have some choice towards the start of the number, three possibilities in fact.

So altogether the five possibilities are:

11111223+
33332221

11111233+
33332211

11122333+
33321111

11222333+
33221111

12222333+
32221111

253.14 – Two Point Oh Eight – Elliott Line

Let’s begin by seeing what happens with four digits, five digits, etc.

With four digits there are two numbers that work: 5252 and 5772.

Since in each case abcd/dcba is equivalent to 52/25, abcd must be a multiple of 52 and dcba must be a multiple of 25. The same multiple in fact.

Let’s take a look at exactly what that multiple is for the first few:

52/25 = 1
572/275 = 11
5252/2525 = 101
5772/2775 = 111
52052/25025 = 1001
57772/27775 = 1111

You might notice that the length of each of these factors is always one less than the number of digits of the final number, because multiplying by 52 or 25 increases the digits by 1. You might also notice that each number is palindromic, because the denominator needs to be the reverse of the numerator. And that it only uses digits 1 and 0, because if you had any digits greater than 1, there would be carrying, which would disrupt the digits of the numerator and the denominator in different ways.

For the 25-digit numbers of the question, we need to find all palindromic 24-digit numbers that are made up of 1s and 0s.

The first digit needs to be a 1 (otherwise the number would not be 25 digits long). The next 11 digits can be either 0 or 1, and the remaining 12 digits are defined by the fact that it needs to be a palindrome.

So we have 11 binary decisions to make. 2^11 = 2048. Therefore there are 2048 25-digit numbers which, when divided by their reversal, equal 2.08. You’ll forgive me for not listing them all, but I’ll pick one at random by flipping a coin:

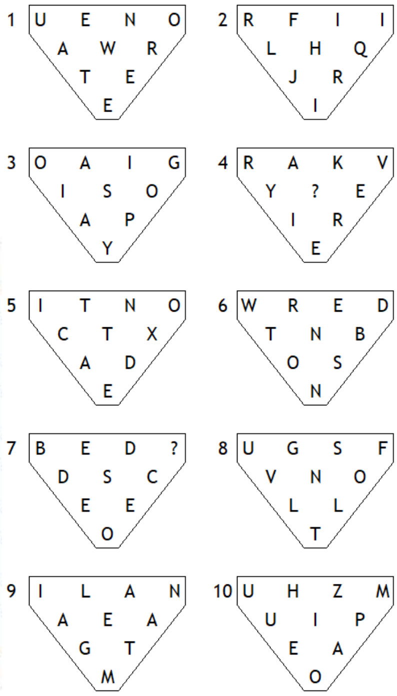
100010110010010011010001:

5200525720520520572520052/2500252750250250275250025 = 2.08

253.15 – Ten Pin Bowling – Elliott Line

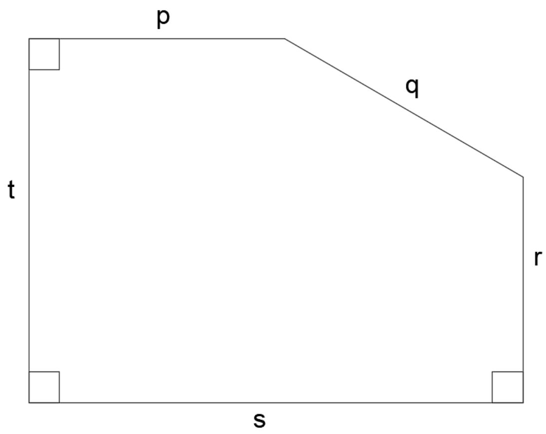
As the letter frames are generated randomly from a scrabble set, I can play along myself. I personally achieved a modest score of 114. I began well with a decent spare on frame 1 but frame 2 was a bit of a horror show. On both frames 4 and 7 I made judicious use of the blank to get spares. On frame 5 I only scored 5+4 but there is a scrabble-legal ten letter word available for a strike: DETOXICANT. I didn’t get that without help, so can’t fairly claim it for my own score. However I did score over 100 so I can be happy with my performance.

FRAME	WORD 1	WORD 2	WORD 1	WORD 2	BONUS
1	outwear	nee	7	3	3
2	fir	hi	3	2	0
3	yoga	soap	4	4	0
4	verity	rake	6	4	5
5	toxic	dent	5	4	0
6	trends	won	6	3	0
7	decodes	bet	7	3	5
8	volts	flung	5	5	7
9	magenta	ail	7	3	5
10	maize	hop	5	3	0
			114		

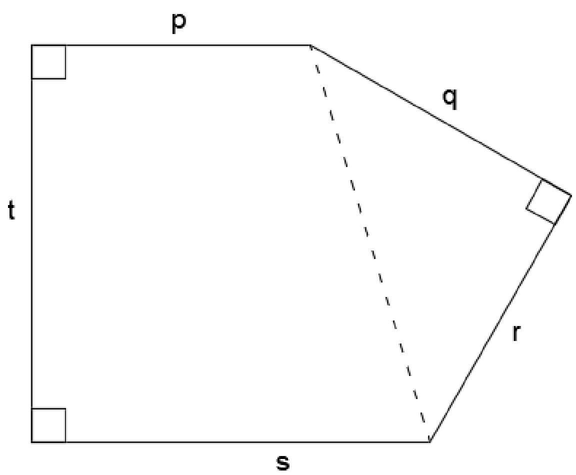


253.16 – Vaguely Defined Pentagon – Elliott Line

The first thing to determine is where on the shape are the three right angles. There are only two possible cases: the two non-right angles are adjacent, or they are not.



Let's consider the case where they are adjacent, as above. Clearly t>r and s>p. The only way this is possible is if q is the second shortest length. q is the hypotenuse of a triangle whose other sides are (s-p) and (t-r). One of these will be equal to 2 and the other 3, and so q equals sqrt(13). This is not a whole number, and so therefore this case is eliminated.



In this second case, the dashed line is simultaneously the hypotenuse of q and r, and of t and (s-p). Checking through the cases where t is the shortest, second shortest, or third shortest side, there are no solutions. I'll demonstrate this by assuming that it is the third shortest. As I've shown s>p, we'll consider q=n, p=n+1, t=n+2, s=n+3, r=n+4.

The square of the dashed line = n^2 + (n+4)^2 = (n+2)^2 + 2^2

Expanding each squared bracket gives:
2n^2+8n+16 = n^2+4n+8

Clearly the left hand side is always greater than the right, for any positive whole number n.

Next let's consider when t is the longest length:
p=n, q=n+1, r=n+2, s=n+3, t=n+4.

The square of the dashed line = (n+1)^2 + (n+2)^2 = 3^2 + (n+4)^2

n^2+2n+1 + n^2+4n+4 = 9 + n^2+8n+16

n^2 – 2n – 20 = 0

Using the quadratic formula, and eliminating the case when n is negative, we get n = 1+sqrt(21). Clearly not a whole number.

Finally let's look at the last remaining case, where t is the second longest side. r=n, q=n+1, p=n+2, t=n+3, s=n+4.

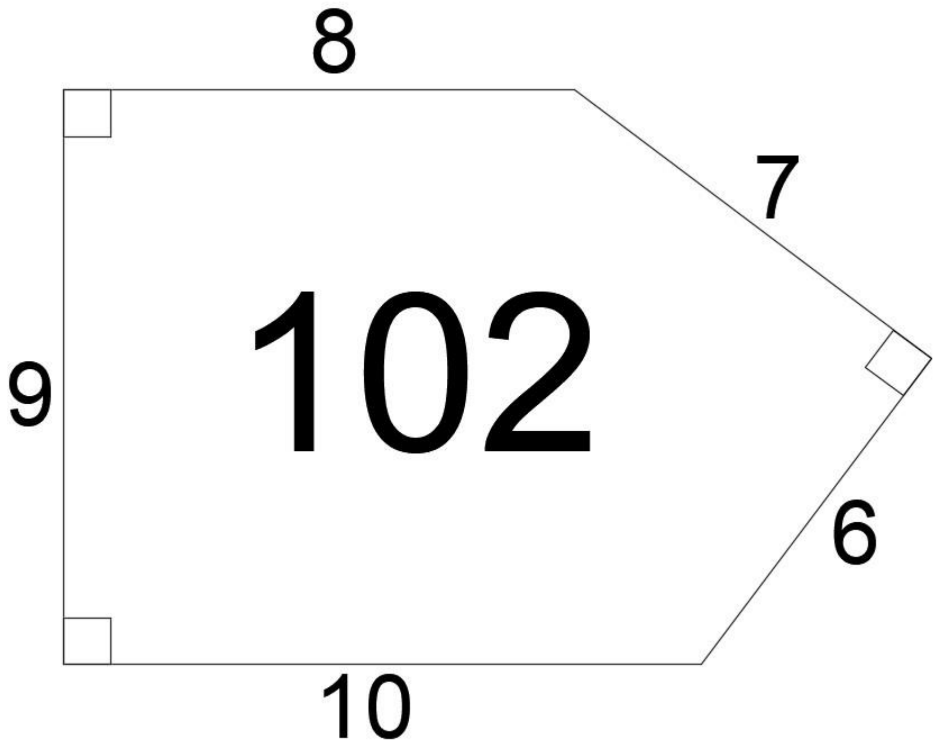
The square of the dashed line = n^2 + (n+1)^2 = (n+3)^2 +2^2

2n^2+2n+1 = n^2+6n+13

n^2 – 4n – 12 = 0

n = (4 + sqrt(64))/2 = 6

So we now know that the side lengths are 6,7,8,9,10 starting at r and going anti-clockwise. Finally, to find the area, we work out the area of the trapezium to the left of the dashed line and the triangle to the right: 81+21 = 102. And that's the answer!



253.17 – Weird Inequalities – Elliott Line

On a clock face!

253.18 – Zipline Futoshiki – Elliott Line

7	>	1	<	2	<	11
∧		∧		∧	✕	∨
16	>	15	>	4	>	3
∨		∨	✕	∧		∧
8	>	5	<	6	<	12
∧	✕	∧		∧	✕	∧
10	>	9	<	13	<	14